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Natural Effective Supersymmetry

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Abstract

Much heavier sfermions of the first-two generations than the other superparticles provide a natural explanation for the flavor and CP problems in the supersymmetric standard model (SUSY SM). However, the heavy sfermions may drive the mass squareds for the light third generation sfermions to be negative through two-loop renormalization group (RG) equations, breaking color and charge. Introducing extra matters to the SUSY SM, it is possible to construct models where the sfermion masses are RG invariant at the two-loop level in the limit of vanishing gaugino-mass and Yukawa-coupling contributions. We calculate the finite corrections to the light sfermion masses at the two-loop level in the models. We find that the finite corrections to the light-squark mass squareds are negative and can be less than $(0.3 - 1)\%$ of the heavy-squark mass squareds, depending on the number and the parameters of the extra matters. We also discuss whether such models realized by the $U(1)_X$ gauge interaction at the GUT scale can satisfy the constraints from Δm_K and ϵ_K naturally. When both the left- and right-handed down-type squarks of the first-two generations have common $U(1)_X$ charges, the supersymmetric contributions to Δm_K and ϵ_K are sufficiently suppressed without spoiling naturalness, even if the flavor-violating supergravity contributions to the sfermion mass matrices are included. When only the right-handed squarks of the first-two generations have a common $U(1)_X$ charge, we can still satisfy the constraint from Δm_K naturally, but evading the bound from ϵ_K requires the CP phase smaller than 10^{-2} .

1 Introduction

The Standard Model (SM) has enjoyed remarkable successes in describing physics down to a scale of 10^{-18} m, the weak scale. However, the quadratically divergent radiative correction to the Higgs boson mass leads to the well-known naturalness problem. Supersymmetry (SUSY) provides a solution to this problem by extending the chiral symmetry to the scalar partners. At present, the minimal supersymmetric standard model (MSSM) is considered to be the most promising candidate beyond the SM [1].

The MSSM is severely constrained by flavor physics. Introduction of SUSY-breaking terms provides new sources of the flavor-changing neutral currents (FCNC's) and CP violation. For instance, the experimental value of the mass difference between K_L and K_S , Δm_K , leads to the upper bound on the squark contribution written as

$$\sin^2 \theta_{\tilde{d}} \left(\frac{\Delta m_{\tilde{d}}^2}{\bar{m}_{\tilde{d}}^2} \right)^2 \left(\frac{30 \text{ TeV}}{\bar{m}_{\tilde{d}}} \right)^2 \lesssim 1, \quad (1)$$

in a basis where the quark mass matrices are diagonal [2]. Here, $\bar{m}_{\tilde{d}}^2$ is the averaged squark mass, and $\Delta m_{\tilde{d}}^2$ and $\sin^2 \theta_{\tilde{d}}$ are the mass-squared difference and the mixing angle between the down-type squarks of the first-two generations. In order to satisfy this constraint, *i*) $\Delta m_{\tilde{d}}^2 \simeq 0$ (degeneracy [3]), *ii*) $\sin \theta_{\tilde{d}} \simeq 0$ (alignment [4]), *iii*) $\bar{m}_{\tilde{d}} \gtrsim 30$ TeV (decoupling [5]), or the hybrid of them is required.

In this article, we discuss the third possibility, the decoupling scenario. In this scenario, the masses for the third generation squarks, Higgsinos, and gauginos are of the order of the weak scale while the other squarks and sleptons are heavy enough to suppress the FCNC and CP-violating processes. This is still natural at the one-loop level, since the squarks and sleptons of the first-two generations are not strongly coupled with the Higgs boson. This mass spectrum is sometimes called as effective supersymmetry or the more minimal supersymmetric standard model. It is realized in the anomalous U(1) SUSY-breaking models [6, 7], U(1)' models [8], composite models of the first-two generation particles [9], or radiatively-driven models with specific boundary conditions [10].

However, this scenario generically suffers from a problem that the third generation sfermions obtain the vacuum expectation value (VEV), breaking color and charge [11, 12]. At the two-loop level the heavy first-two generation sfermions contribute to the the renormalization group (RG) equations for the third generation sfermion masses through the gauge interactions. The RG equations for the SUSY-breaking sfermion masses \tilde{m}_r^2 through the SM gauge interactions are given by [13]

$$\begin{aligned} \mu \frac{d\tilde{m}_r^2}{d\mu} = & \sum_A 8 \left(\frac{\alpha_A}{4\pi} \right)^2 C_r^A \sum_s T_s^A \tilde{m}_s^2 \\ & + 2 \left(\frac{\alpha_Y}{4\pi} \right) Y_r \sum_s Y_s \tilde{m}_s^2 + \sum_A 8 \left(\frac{\alpha_Y}{4\pi} \right) \left(\frac{\alpha_A}{4\pi} \right) Y_r \sum_s Y_s C_s^A \tilde{m}_s^2, \end{aligned} \quad (2)$$

at the two-loop level in the $\overline{\text{DR}}'$ scheme [14]. Here, we have taken a limit where the gaugino masses vanish. An index A ($= 1 - 3$) represents the SM gauge groups, and we have adopted

the $SU(5)_{\text{GUT}}$ normalization for the $U(1)_Y$ gauge coupling ($\alpha_1 \equiv (5/3)\alpha_Y$). T_r^A , C_r^A and Y_r are the Dynkin index, the quadratic Casimir coefficient and the hypercharge for the sfermion r , respectively. We find that all sfermion mass squareds contribute to the RG equations at the two-loop level. Thus, the contribution from the heavy sfermions may destroy the mass spectrum by driving the light-sfermion mass squareds to be negative at low energies. This fact makes it difficult to construct models where such a hierarchical sfermion mass spectrum is generated at much higher energy scale than the weak scale.

However, the hierarchical mass spectrum can be stabilized against the RG evolution if the following relations among the SUSY-breaking mass squareds for the heavy sfermions are imposed [15]:

$$\sum_r T_r^A \tilde{m}_r^2 = 0, \quad (3)$$

$$\sum_r Y_r \tilde{m}_r^2 = 0, \quad (4)$$

$$\sum_r Y_r C_r^A \tilde{m}_r^2 = 0, \quad (5)$$

for $A = 1 - 3$. These relations cannot be satisfied by the heavy MSSM sfermions alone, since all of them have positive SUSY-breaking mass squareds. However, the relations can be satisfied if we introduce extra fields with negative SUSY-breaking mass squareds which transform nontrivially under the SM gauge groups, since the sum is taken over the extra fields as well as the heavy MSSM sfermions. The extra fields should have the supersymmetric masses to avoid the breaking of the SM gauge groups caused by their condensations. Thus, the extra fields have to be in vector-like representations of the SM gauge groups, and we call them *extra matters*. The supersymmetric masses for the extra matters should not be much larger than the SUSY-breaking scalar masses. Otherwise, the large radiative correction is generated below the energy scale where the extra matters are decoupled. We refer this extension of the MSSM to the Natural Effective SUSY SM (NESSM), hereafter.

The NESSM can be realized by assuming that the SUSY-breaking masses for the heavy sfermions and extra matters are generated by a D -term VEV of some $U(1)_X$ gauge interaction such that $\tilde{m}_r^2 = Q_r^X \langle -D_X \rangle$. The $U(1)_X$ should be broken in order to give the supersymmetric masses for the extra matters. In terms of the $U(1)_X$ charges, Eqs. (3, 4, 5) are written as

$$\sum_r T_r^A Q_r^X = 0, \quad (6)$$

$$\sum_r Y_r Q_r^X = 0, \quad (7)$$

$$\sum_r Y_r C_r^A Q_r^X = 0. \quad (8)$$

The first equation corresponds to vanishing mixed anomalies between $U(1)_X$ and the SM gauge groups. The second and third ones mean that $U(1)_X$ has no kinetic mixing with $U(1)_Y$ at the one- and two-loop levels. If $U(1)_X$ is anomaly-free and all fields charged under $U(1)_X$ are embedded in the $SU(5)_{\text{GUT}}$ multiplets, Eqs. (6, 7) are satisfied automatically. In this case, we

have only to choose the $U(1)_X$ charges for the extra matters to satisfy Eq. (8). Even if $U(1)_X$ is anomalous, however, we can still construct a model in which some of the fields are decoupled at the $U(1)_X$ breaking scale and Eqs. (6, 7, 8) are satisfied at low energies.

In this article, we calculate the finite corrections to the light-sfermion mass squareds at the two-loop level in the NESSM, assuming that all fields are embedded in the $SU(5)_{\text{GUT}}$ multiplets at the GUT scale. In the NESSM, while the dangerous contributions which lead to color and charge breaking do not exist in the RG equations for the SUSY-breaking scalar masses, the finite corrections from the heavy sfermions and the extra matters at the two-loop level may still drive the light-sfermion mass squareds to be negative. We find that the finite corrections to the light-squark mass squareds depend on the number and the parameters of the extra matters and they are less than $(0.3 - 1)\%$ of the heavy-squark mass squareds.

We also discuss whether the models in which the NESSM is realized by the $U(1)_X$ gauge interaction at the GUT scale ($M_{\text{GUT}} \simeq 2 \times 10^{16}$ GeV) are viable or not in the light of the experimental constraints. The anomalous $U(1)$ SUSY-breaking model given in Ref. [15] is an explicit example for such models. On this setup, the supergravity contributions to the SUSY-breaking terms, which are generically non-universal in flavor space, are suppressed compared with the $U(1)_X D$ -term contribution. Moreover, the breaking of the $U(1)_X$ symmetry can naturally explain the hierarchical structure of the quark and lepton masses by the Froggatt-Nielsen mechanism [16]. In this paper, we consider the constraints from Δm_K and ϵ_K , which lead to the severest bound on the heavy-sfermion mass scale. We find that, when both the left- and right-handed down-type squarks (only the right-handed squarks) of the first-two generations have common $U(1)_X$ charges, the contributions to Δm_K and ϵ_K (Δm_K) are sufficiently suppressed without spoiling naturalness even if the flavor-violating supergravity contributions to the sfermion mass matrices are included.

This paper is organized as follows. In the next section, we show our setup and the $U(1)_X$ charge assignments. We explain that the squark and slepton masses are inversely related to the quark and lepton masses through their $U(1)_X$ charges. In Section 3, we derive the formula for the finite corrections to the light sfermion masses at the two-loop level in the NESSM, and evaluate them numerically. In Section 4, using the constraints from Δm_K and ϵ_K the lower bounds on the third generation sfermion masses at the GUT scale are derived. We neglect the effects of the Yukawa interactions in Section 3 and 4, since they are model-dependent. The effects are evaluated in Section 5. We also discuss the three-loop RG contributions to the light sfermion masses there. Section 6 is devoted to the conclusions and discussion. In Appendix A, we review the anomalous $U(1)$ SUSY-breaking model given in Ref. [15] as one of the explicit realizations of the NESSM. In Appendix B, we show the complete formulae for the contributions to the light sfermion masses from the heavy ones at the two-loop level. The formulae for Δm_K and ϵ_K are given in Appendix C.

2 Setup

In this section, we show the setup of the NESSM on which we perform our analyses in later sections. In the NESSM, the SUSY-breaking scalar masses for the heavy sfermions and extra scalars are given by the $U(1)_X$ D -term. We here assume that the nonzero $U(1)_X$ D -term is generated associated with the breaking of the $U(1)_X$ gauge symmetry caused by the VEV of a Φ field which has a $U(1)_X$ charge of -1 ($Q_\Phi^X = -1$). In this case, the $U(1)_X$ D -term is related to the F -term of the Φ field at a (local) minimum of the potential as follows:

$$-D_X = \frac{g_X^2}{M_X^2} | -F_\Phi |^2, \quad (9)$$

where M_X ($= g_X |\langle \Phi \rangle|$) is the $U(1)_X$ gauge boson mass. Here, we have assumed that the VEV's and F -terms of the other fields are negligibly small for simplicity. Then, the relation between the supergravity contribution m_0 and the $U(1)_X$ D -term ($m_D \equiv \sqrt{| -D_X |}$) is given by

$$m_0 \simeq \frac{| -F_\Phi |}{M_{\text{pl}}} = \frac{|\langle \Phi \rangle|}{M_{\text{pl}}} m_D, \quad (10)$$

where M_{pl} is the reduced Planck scale ($M_{\text{pl}} \simeq 2 \times 10^{18}$ GeV). This shows that the supergravity contributions to the sfermion masses are, in general, suppressed compared with the D -term contribution, m_D , as long as $|\langle \Phi \rangle|$ is smaller than M_{pl} . Then, the $U(1)_X$ charged sfermions have large masses of the order of the $U(1)_X$ D -term, m_D , while the neutral sfermions only receive smaller masses of order m_0 from the supergravity contributions. In order to obtain desired mass spectrum, we set m_0 to be of the order of the weak scale and $|\langle \Phi \rangle|/M_{\text{pl}} \simeq (10^{-1} \sim 10^{-2})$.

Next, we consider the $U(1)_X$ charge assignment. We assume that all the fields of the NESSM are embedded in $SU(5)_{\text{GUT}}$ multiplets at the GUT scale. It not only maintains the successful gauge coupling unification but also guarantees vanishing $U(1)_Y$ D -term contribution at the one-loop level (Eq. (7)). We also assume that one pair of the extra matters are introduced in $(\mathbf{5} + \mathbf{5}^*)$ representation of the $SU(5)_{\text{GUT}}$ for simplicity. The extension to the other cases is straightforward. Then, the $U(1)_X$ charges for the $\mathbf{5}$ and $\mathbf{5}^*$ extra matters, $Q_{\mathbf{5}_{\text{ex}}}^X$ and $Q_{\mathbf{5}^*_{\text{ex}}}^X$, are determined from those for the quarks and leptons in the SM to satisfy Eqs. (6, 8) as

$$Q_{\mathbf{5}_{\text{ex}}}^X = -2 \sum_{i=1}^3 Q_{\mathbf{10}_i}^X, \quad (11)$$

$$Q_{\mathbf{5}^*_{\text{ex}}}^X = - \sum_{i=1}^3 (Q_{\mathbf{10}_i}^X + Q_{\mathbf{5}^*_i}^X), \quad (12)$$

where $Q_{\mathbf{10}_i}^X$ and $Q_{\mathbf{5}^*_i}^X$ ($i = 1 - 3$) are the $U(1)_X$ charges for the quarks and leptons embedded in the $\mathbf{10}$ and $\mathbf{5}^*$ representations of the $SU(5)_{\text{GUT}}$, respectively. An index i represents the generation.

The $U(1)_X$ charges for the SM matter multiplets are determined by the following considerations. The naturalness argument tells us that the superparticles which are strongly coupled

with the Higgs boson are necessary to have masses around the weak scale and thus should have zero $U(1)_X$ charges. This requires that $Q_{\mathbf{10}_3}^X = 0$, since the top squarks and left-handed bottom squark are coupled with the Higgs boson through large top Yukawa coupling. On the other hand, the superpartners of the light quarks and leptons have to be heavy enough to sufficiently suppress the FCNC processes, so that they should have positive $U(1)_X$ charges. This explains the hierarchy of the quark and lepton masses naturally by the Froggatt-Nielsen mechanism, since the Yukawa matrices are generated through the VEV of the Φ field suppressed by suitable powers of $\langle \Phi \rangle / M_{\text{pl}}$ [16] as

$$W = \sum_{i,j=1}^3 (f_u)_{ij} \left(\frac{\langle \Phi \rangle}{M_{\text{pl}}} \right)^{Q_{\mathbf{10}_i}^X + Q_{\mathbf{10}_j}^X} \Psi_{\mathbf{10}_i} \Psi_{\mathbf{10}_j} H_u \\ + \sum_{i,j=1}^3 (f_d)_{ij} \left(\frac{\langle \Phi \rangle}{M_{\text{pl}}} \right)^{Q_{\mathbf{10}_i}^X + Q_{\mathbf{5}^*_j}^X} \Psi_{\mathbf{10}_i} \Psi_{\mathbf{5}^*_j} H_d, \quad (13)$$

where H_u and H_d are the Higgs doublets for which we have assumed the vanishing $U(1)_X$ charges. Thus, it is possible to reproduce the observed quark and lepton mass matrices with $(f_u)_{ij}, (f_d)_{ij} = O(1)$ if we appropriately choose the $U(1)_X$ charges $Q_{\mathbf{10}_i}^X$ and $Q_{\mathbf{5}^*_j}^X$ [17].

According to the above arguments, we consider the following $U(1)_X$ charge assignments:

Model	I	II	III	IV
$Q_{\mathbf{10}_1}^X$	1	2	2	2
$Q_{\mathbf{10}_2}^X$	1	1	1	1
$Q_{\mathbf{10}_3}^X$	0	0	0	0
$Q_{\mathbf{5}^*_1}^X$	1	1	1	2
$Q_{\mathbf{5}^*_2}^X$	1	1	1	1
$Q_{\mathbf{5}^*_3}^X$	0	1	0	1
$Q_{\mathbf{5}^*_{\text{ex}}}^X$	-4	-6	-6	-6
$Q_{\mathbf{5}^{*\text{ex}}}^X$	-4	-6	-5	-7

Here, the $U(1)_X$ charges for the Higgs multiplets are zero in all the models and the charges for the extra-matter multiplets were determined by Eqs. (11, 12). Model (I) is the simplest possibility realizing the decoupling scenario. In this model, the FCNC's are strongly suppressed due to the degeneracy of the SUSY-breaking masses between the first-two generation sfermions, while the hierarchy among the quark and lepton masses cannot be explained completely. Models (II-IV) are motivated to explain the fermion mass hierarchy by the Froggatt-Nielsen mechanism. In particular, in Models (II, IV) the second and third generation doublet leptons have the same $U(1)_X$ charges, $Q_{\mathbf{5}^*_2}^X = Q_{\mathbf{5}^*_3}^X$, so that the observed large mixing between ν_μ and ν_τ [18] is naturally explained.¹ In these models, the FCNC processes are less suppressed than in Model (I). Models (II, III) have non-degenerate SUSY-breaking masses for the left-handed down-type squarks in the first-two generations. Thus, K^0 - \bar{K}^0 oscillation has dominant contribution

¹ Models (II) and (IV) prefer large and small angle MSW solutions to the solar neutrino problem, respectively [19, 20].

from the off-diagonal elements in the left-handed squark mass matrix. Model (IV) has non-degenerate SUSY-breaking masses for both the left- and right-handed down-type squarks in the first-two generations, so that K^0 - \bar{K}^0 oscillation receives contributions from off-diagonal elements of both the left- and right-handed squark mass matrices. As a result, Model (IV) is more severely constrained from K^0 - \bar{K}^0 mixing than Models (II, III), as will be discussed later.

We now discuss the other SUSY-breaking parameters. Since the gaugino masses are constrained by naturalness argument, we consider them to be of the order of the weak scale. Indeed, in the anomalous U(1) SUSY-breaking model given in Appendix A, the gaugino masses arise from the F -term of the dilaton field [21] and their sizes can be of the order of the weak scale [22]. We treat the gaugino mass (at the GUT scale) as a free parameter in the phenomenological analyses below. Also, the superpotential Eq. (13) generates the SUSY-breaking trilinear scalar couplings,

$$(A_u)_{ij} = (Q_{\mathbf{10}_i}^X + Q_{\mathbf{10}_j}^X) \frac{F_\Phi}{\langle \Phi \rangle}, \quad (14)$$

$$(A_d)_{ij} = (Q_{\mathbf{10}_i}^X + Q_{\mathbf{5}^*_j}^X) \frac{F_\Phi}{\langle \Phi \rangle}. \quad (15)$$

The $(A_u)_{ij}$ and $(A_d)_{ij}$ are defined by $\mathcal{L}_{\text{soft}} = (y_u)_{ij}(A_u)_{ij}\tilde{\psi}_{\mathbf{10}_i}\tilde{\psi}_{\mathbf{10}_j}h_u + (y_d)_{ij}(A_d)_{ij}\tilde{\psi}_{\mathbf{10}_i}\tilde{\psi}_{\mathbf{5}^*_j}h_d$. The $(y_u)_{ij}$ and $(y_d)_{ij}$ are the Yukawa couplings $W = (y_u)_{ij}\Psi_{\mathbf{10}_i}\Psi_{\mathbf{10}_j}H_u + (y_d)_{ij}\Psi_{\mathbf{10}_i}\Psi_{\mathbf{5}^*_j}H_d$, and $\tilde{\psi}$ and h represent the scalar components of the corresponding supermultiplets Ψ and H , respectively. From Eq. (10), we find that $A_u \simeq A_d \simeq m_D$ except for the ones which involve only the light sfermions.

The supersymmetric masses and the holomorphic SUSY-breaking masses for the extra matters come from the $U(1)_X$ symmetry breaking. The supersymmetric masses should be of the same order with m_D as we mentioned in Introduction. While these features are naturally explained in the explicit model given in Ref. [15] (see Appendix A), we mainly adopt a phenomenological approach in this article and take the masses as input parameters. We parameterize the supersymmetric and holomorphic SUSY-breaking mass parameters as

$$W_{\text{ex}} = m_\psi \Psi_{\mathbf{3}_{\text{ex}}} \Psi_{\mathbf{3}^*_{\text{ex}}} + m'_\psi \Psi_{\mathbf{2}_{\text{ex}}} \Psi_{\mathbf{2}^*_{\text{ex}}}, \quad (16)$$

$$\mathcal{L}_{\text{ex, soft}} = -(F_\psi \tilde{\psi}_{\mathbf{3}_{\text{ex}}} \tilde{\psi}_{\mathbf{3}^*_{\text{ex}}} + F'_\psi \tilde{\psi}_{\mathbf{2}_{\text{ex}}} \tilde{\psi}_{\mathbf{2}^*_{\text{ex}}} + \text{h.c.}). \quad (17)$$

Here, the triplet extra matters $\Psi_{\mathbf{3}_{\text{ex}}}$ and $\Psi_{\mathbf{3}^*_{\text{ex}}}$ and the doublet extra matters $\Psi_{\mathbf{2}_{\text{ex}}}$ and $\Psi_{\mathbf{2}^*_{\text{ex}}}$ are embedded in the $\Psi_{\mathbf{5}_{\text{ex}}}$ and $\Psi_{\mathbf{5}^*_{\text{ex}}}$. In this article, we impose the $SU(5)_{\text{GUT}}$ relations on the supersymmetric and the holomorphic SUSY-breaking masses, $m_\psi = m'_\psi$ and $F_\psi = F'_\psi$, at the GUT scale unless otherwise stated. Then, the masses for the heavy sfermions and extra matters are completely determined by three free parameters m_D , m_ψ and F_ψ once a $U(1)_X$ charge assignment is specified.

3 Corrections to the Light Sfermions

In the NESSM, the two-loop RG contributions to the light sfermions are canceled between the heavy sfermions and the extra-matter multiplets. However, there are still negative finite corrections. In this section, we numerically estimate the effects of the finite corrections in various cases, using their explicit form which is completely calculated at the two-loop level in Appendix B. We conclude this section by showing how hierarchical the sfermion mass spectrum can be in the NESSM without breaking color and charge, comparing the NESSM with the original effective SUSY in which there is no extra-matter multiplet.

Now, let us numerically estimate the finite corrections in the models discussed in the previous section. We first consider Models (I, II). In these models, the $U(1)_X$ charge assignments are consistent with the $SU(5)_{\text{GUT}}$ and the extra-matter multiplets have an invariance under the parity $\Psi_{5_{\text{ex}}} \leftrightarrow \Psi_{5^*_{\text{ex}}}$, so that the finite corrections to the light sfermions, $m_{f,\text{finite}}^2$, take relatively simple forms. The explicit expressions for $m_{f,\text{finite}}^2$ are given in Eqs. (117, 118, 119) in Appendix B. These expressions can be further simplified in the case of one-pair extra-matter multiplets as follows:

$$\begin{aligned} m_{f,\text{finite}}^2 &= 4 \sum_A \left(\frac{\alpha_A}{4\pi} \right)^2 C_f^A \left\{ -\text{Tr}_{\tilde{F}} \left[T_{\tilde{F}}^A m_{\tilde{F}}^2 \left(\log \frac{m_\psi^2}{m_{\tilde{F}}^2} + \frac{1}{3}\pi^2 \right) \right] + \frac{1}{2} m_\psi^2 G(y_1, y_2) \right\} \\ &\quad + 4 \sum_A \left(\frac{\alpha_Y}{4\pi} \right) \left(\frac{\alpha_A}{4\pi} \right) Y_f \text{Tr}_{\tilde{F}} \left[Y_{\tilde{F}} C_{\tilde{F}}^A m_{\tilde{F}}^2 \log m_{\tilde{F}}^2 \right], \end{aligned} \quad (18)$$

where

$$G(y_1, y_2) = \left(y_1 \log y_1 - 2y_1 \text{Li}_2(1 - \frac{1}{y_1}) + \frac{1}{2} y_1 \text{Li}_2(1 - \frac{y_2}{y_1}) \right) + (y_1 \leftrightarrow y_2), \quad (19)$$

$$y_1 \equiv \frac{m_1^2}{m_\psi^2}, \quad y_2 \equiv \frac{m_2^2}{m_\psi^2}, \quad (20)$$

where \tilde{F} denotes the heavy sfermions; m_1 and m_2 are the mass eigenvalues for the extra scalars (see Appendix B). Here, we have neglected the difference between the supersymmetric masses (and the holomorphic SUSY-breaking masses) of the triplet and doublet extra matters for demonstrational purpose, though it will be included in later numerical calculations.

The first term of the first line in Eq. (18) gives dominant contributions to the light-sfermion mass squareds. They are negative and could cause color and charge breaking since $m_\psi > m_{\tilde{F}}$. The second term has the same form as the scalar mass squared generated by integrating out the messenger fields in the gauge-mediated SUSY breaking mechanism [23]. Therefore, it gives the positive contributions almost proportional to the SUSY-breaking bilinear coupling F_ψ of the extra scalars. Even if $F_\psi = 0$, however, $G(y_1, y_2)$ remains positive, since m_1^2 and m_2^2 are smaller than m_ψ^2 by the $U(1)_X$ D -term. The last term which is generated through $U(1)_Y$ D -term does not give significant contributions due to the smallness of the hypercharge gauge coupling. The renormalization-point dependence of the term is canceled out due to Eq. (8).

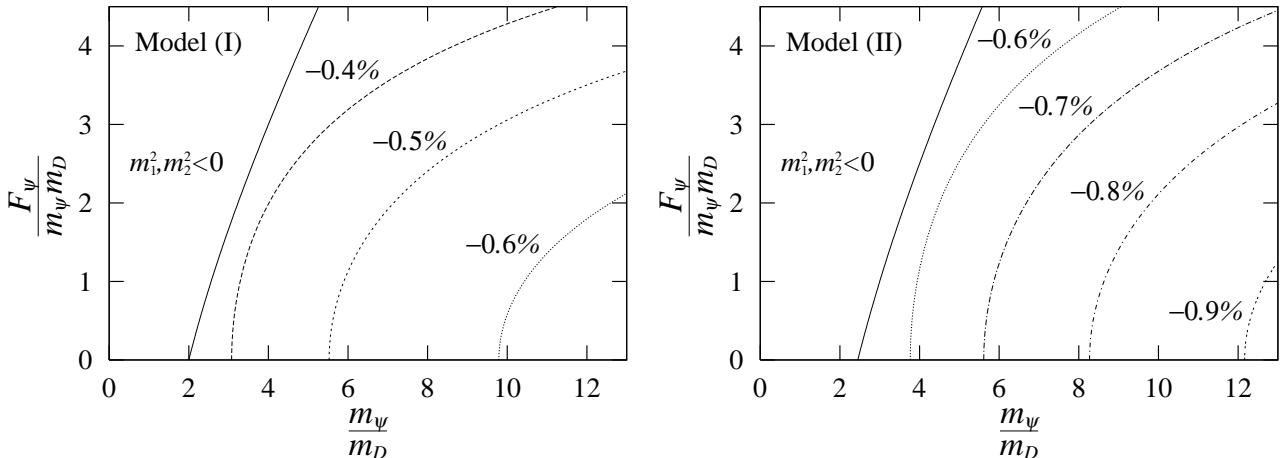


Figure 1: Finite corrections to the light squarks through the strong interaction. The percentages on the figures represent the ratio of the finite corrections to the heavy-sfermion mass scale, $m_{\tilde{q}, \text{finite}}^2/m_D^2$. The left figure is in Model (I) and the right is in Model (II). The left side of the solid line is excluded since the mass squared for the extra scalar is negative and color is broken. Here, we have taken $m_D = 10$ TeV and evaluated all the parameters at $\mu = 10$ TeV.

In order to see the dependence of the finite corrections on m_ψ , m_D and F_ψ , we fix the heavy-sfermion mass scale $m_D = 10$ TeV as an overall scale and take m_ψ/m_D and $F_\psi/(m_\psi m_D)$ as remaining two dimensionless free parameters. The ratio m_ψ/m_D must be of order unity in the NESSM, since otherwise the heavy sfermions generate the large radiative corrections below the scale m_ψ where the extra-matter multiplets decouple. Then, $F_\psi/(m_\psi m_D)$ should not be much larger than one to avoid the negative mass squared for the extra scalar. Indeed, $F_\psi/(m_\psi m_D)$ is of order unity in the explicit example constructed.

Fig. 1 illustrates the dependence of the finite corrections on these two parameters. As an example, we consider the corrections to the light squarks in Models (I, II) and plot the ratio of the finite corrections to the heavy sfermion mass-scale squared m_D^2 . Here, we only take into account the dominant two-loop contribution through the strong interaction, that is we drop the last term and set $A = 3$ and $C_{\tilde{f}}^{(3)} = 4/3$ in Eq. (18). The minimum of the ratio is -0.36% ($m_\psi/m_D = 2.2$) and -0.54% ($m_\psi/m_D = 2.7$) in Models (I) and (II), respectively, when $F_\psi = 0$. The difference between Models (I) and (II) in Fig. 1 comes mainly from the group-theoretical factor in Eq. (18), $\text{Tr}_{\tilde{F}} [T_{\tilde{F}}^A m_{\tilde{F}}^2] / m_D^2 = \text{Tr}_{\tilde{F}} [T_{\tilde{F}}^A Q_{\tilde{F}}^X]$, which is 4 in Model (I) and 6 in Model (II). Thus, the corrections in Model (II) are larger than in Model (I) approximately by a factor of 1.5.

The ratio given in Fig. 1 determines how hierarchical the sfermion mass spectrum can be without breaking color and charge. Since the bare mass $m_{\tilde{q},0}$ for the light squarks has to be larger than these negative corrections, it gives a lower bound on $m_{\tilde{q},0}$. In Model (I), for example, we obtain

$$m_{\tilde{q},0}^2 \geq -m_{\tilde{q}, \text{finite}}^2 \simeq 0.4\% \times m_D^2, \quad (21)$$

when the supersymmetric mass for the extra matter, m_ψ , is not much larger than m_D as discussed in the previous section.² Since $m_{\tilde{q},0}$ set the scale for the light sfermion masses, we find that the splitting between the heavy and light sfermions cannot far exceed an order of magnitude. In the case of the original effective SUSY models (models without extra-matter multiplets), the RG contributions to the light sfermions, $m_{f,\log}^2$, are written by solving the RG equation Eq. (2) as follows:

$$m_{f,\log}^2(\mu) = 4 \sum_A \frac{C_f^A}{b_A} \text{Tr}_{\tilde{F}} \left[T_{\tilde{F}}^A m_{\tilde{F}}^2 \right] \left(\frac{\alpha_A(\mu)}{4\pi} - \frac{\alpha_{\text{GUT}}}{4\pi} \right), \quad (22)$$

where b_A is the coefficient for the one-loop beta function of the SM gauge coupling and α_{GUT} is the gauge coupling constant at the GUT scale. Here, the contributions from the $U(1)_Y$ D -term are neglected. In Model (I), for example, radiative corrections to the light squarks through the strong interaction are estimated as

$$m_{\tilde{q},\log}^2(\mu = 10 \text{ TeV}) \simeq -2.3\% \times m_D^2, \quad (23)$$

so that the bare light-squark mass squared $m_{\tilde{q},0}^2$ should be larger than about $-m_{\tilde{q},\log}^2(\mu = 10 \text{ TeV}) \simeq 2.3\% \times m_D^2$. The rough estimations in Eqs. (21, 23) show that the splitting between the heavy and light sfermion masses in the NESSM can be more than twice larger than in the original effective SUSY. This conclusion remains true even if full radiative corrections are taken into account, as will be shown below.

We here comment on the effect of introducing more than one pairs of extra-matter multiplets. If we introduce more extra-matter pairs, the extra matters could have smaller $U(1)_X$ charges satisfying Eqs. (6, 8) and thus smaller supersymmetric masses. Thus, we can reduce the finite corrections to the light-sfermion mass squareds by introducing more extra matters, since it is the supersymmetric mass for the extra matters that determines the size of the negative finite corrections. For example, the minimum size of the finite corrections with $F_\psi = 0$ in Model (I) are reduced to -0.36% , -0.30% , -0.26% , and -0.24% by introducing one, two, three and four pairs of $\Psi_{5_{\text{ex}}}$ and $\Psi_{5^*_{\text{ex}}}$, respectively. However, introducing extra matters changes the beta functions, so that the behavior of the gauge couplings becomes worse at high energy in this case. In this article, we limit ourselves to the case of one pair of $\Psi_{5_{\text{ex}}}$ and $\Psi_{5^*_{\text{ex}}}$, hereafter.

We now include the effects from the gauginos. Since the gauginos give positive RG contributions at the one-loop level, they relax the lower bounds on the light sfermion masses. The lower bounds on the light sfermion masses can be determined as follows. We first set the boundary conditions at the GUT scale 2×10^{16} GeV: we, for simplicity, take the universal gaugino mass $m_{1/2}$ and give the bare mass m_0 to the light sfermions at the GUT scale. Then, we run all the soft SUSY-breaking masses using two-loop RG equations. We neglect the effects of the Yukawa

² In the region where $m_\psi/m_D \gtrsim O(10)$ and $F_\psi/(m_\psi m_D) = O(10)$, the finite corrections can be much smaller due to the accidental cancellation between the negative contribution from the heavy-sfermion loops and the gauge-mediated contribution from the extra-matter multiplets. However, this does not necessarily mean that more hierarchical superparticle mass spectrum can be realized, since the gluino mass receives the correction from the extra-matter loops ($m_{\tilde{g}} \simeq (\alpha_3/4\pi)(F_\psi/m_\psi) \sim 0.1m_D$).

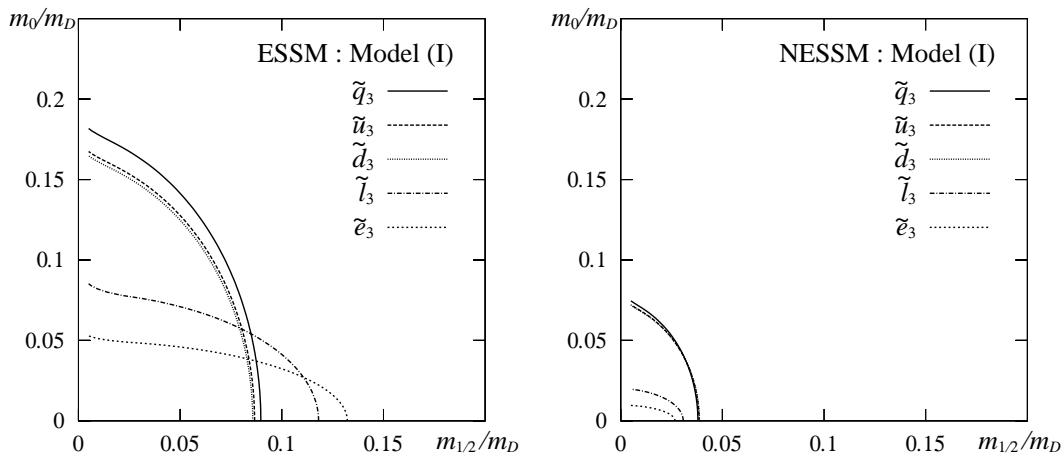


Figure 2: Lower bounds on the mass ratios of the light sfermions to the heavy sfermions in Model (I). m_0 and $m_{1/2}$ represent the light sfermion and gaugino masses at the GUT scale, respectively. The regions below the curves are excluded, since the light sfermions indicated by various lines have negative mass squareds at the weak scale, breaking color and charge. Here, we have set $m_D = 10$ TeV and $m_\psi = 2.0 \times m_D$ at the GUT scale, and neglected the Yukawa couplings.

couplings, since they are rather model-dependent without the assumption of universal scalar masses. The effects will be discussed in Section 5. The heavy sfermions are decoupled at the scale $m_D = 10$ TeV, where the full finite corrections are added to the light sfermions. The positive contributions from the gauginos are included from the GUT scale to the scale where the gluino decouples. After these steps, if any light sfermions have negative mass squareds at the weak scale, the parameter region is excluded and we need larger value of m_0 at the GUT scale.

In Fig. 2 and Fig. 3, we have plotted the lower bounds on the light sfermion masses m_0 as a function of the gaugino mass $m_{1/2}$ in Models (I) and (II). The m_0 and $m_{1/2}$ are the GUT-scale values and normalized by the heavy-sfermion mass scale m_D . In each figure, the left graph (ESSM) is in the case of the original Effective SUSY SM (without extra-matter multiplets) and the right one (NESSM) is of the Natural Effective SUSY SM. We have taken the supersymmetric mass m_ψ for the extra-matter multiplets as $m_\psi = 2.0 \times m_D$ in Model (I) and $m_\psi = 2.5 \times m_D$ in Model (II) at the GUT scale. The various curves represent the lower bounds on the bare masses for the indicated light sfermions, below which they have negative mass squareds at the weak scale. From Figs. 2 and 3 we find that in the NESSM the light sfermion masses and/or the gaugino masses can take values less than half in the ESSM without breaking color and charge. This in turn means that we can obtain more hierarchical mass spectrum in the NESSM than in the ESSM.

Note that we have assumed that the triplet and doublet extra matters have the same supersymmetric masses at the GUT scale. Then, at low energy the supersymmetric mass for the triplet extra matter becomes about twice as large as that for the doublet one due to the

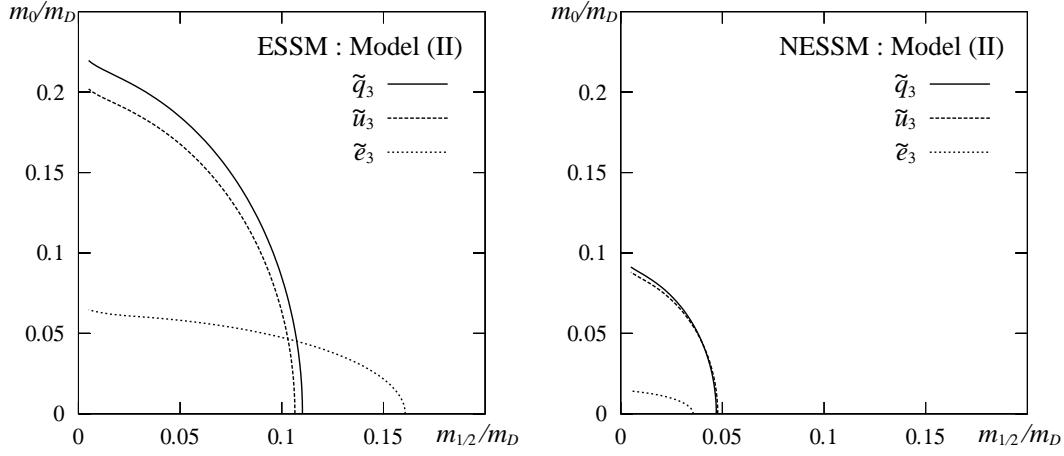


Figure 3: Lower bounds on the mass ratios of the light sfermions to the heavy sfermions in Model (II). We have set $m_D = 10$ TeV and $m_\psi = 2.5 \times m_D$ at the GUT scale.

RG evolution. As a result, we cannot take the supersymmetric mass which minimizes the finite corrections in Fig. 1, since then the doublet extra scalar has negative mass squared. Thus, if we allow different supersymmetric masses for the triplet and doublet extra matters at the GUT scale, we can further reduce the size of finite corrections.³ In this case, however, the running masses for the triplet extra scalars take negative values at the high-energy scale, which means that the scalar potential has another minimum at the nonzero values of the triplet extra scalars.

In the rest of this section, we consider Models (III, IV). The finite corrections in these models are given by Eqs. (114, 115, 116) in Appendix B. They take less simple forms due to the absence of the parity between the extra-matter multiplets, $\Psi_{5_{\text{ex}}} \leftrightarrow \Psi_{5^*_{\text{ex}}}$. In Fig. 4 and Fig. 5, we have plotted the lower bounds on m_0 in Models (III) and (IV), respectively. We have taken the supersymmetric mass for the extra-matter multiplet as $m_\psi = 2.5 \times m_D$ in Model (III) and $m_\psi = 2.7 \times m_D$ in Model (IV) at the GUT scale. All the other parameters have been taken the same as in Models (I, II).

The main difference of Models (III, IV) from Models (I, II) is the existence of the one-loop $U(1)_Y$ D -term,

$$m_{\tilde{f}, \text{FI-1loop}}^2 = \left(\frac{\alpha_Y}{4\pi} \right) Y_{\tilde{f}} \text{Tr}_R \left[Y_R \cos 2\theta \left(m_1^2 \log m_1^2 - m_2^2 \log m_2^2 \right) \right], \quad (24)$$

where R denotes the extra matters. This term arises because the RG evolution from the GUT to low-energy scale splits the supersymmetric masses for the triplet and doublet extra matters. However, the corrections Eq. (24) do not dominate the two-loop contributions, since the hypercharge gauge coupling is much smaller than the strong coupling and there is no enhancement by factors such as 4 or $\text{Tr}_{\tilde{F}} [T_{\tilde{F}}^A m_{\tilde{F}}^2]$ in Eq. (18). As a consequence, Figs. 4 and

³The different supersymmetric masses for the triplet and doublet extra matters do not necessarily contradict with the GUT, since we can make them split using GUT-breaking VEV such as $\langle \Sigma(\mathbf{24}) \rangle$.

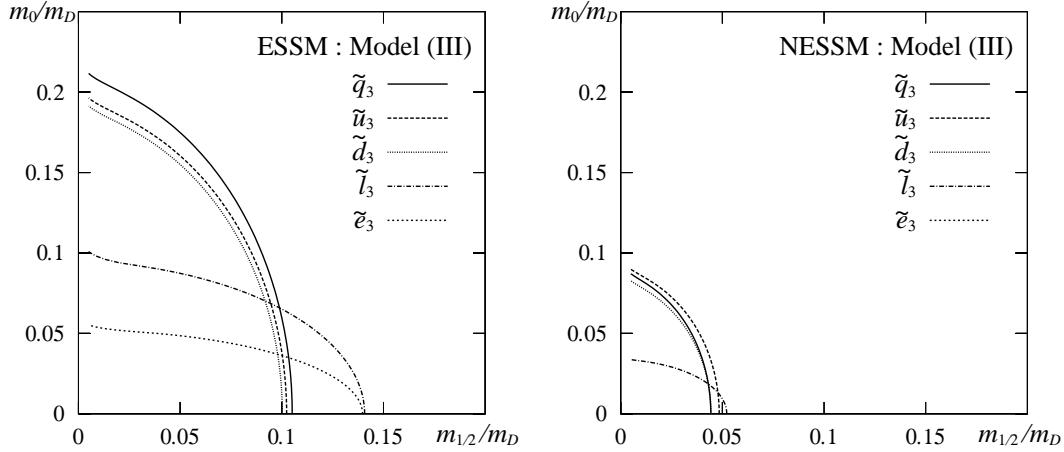


Figure 4: Lower bounds on the mass ratios of the light sfermions to the heavy sfermions in Model (III). We have set $m_D = 10$ TeV and $m_\psi = 2.5 \times m_D$ at the GUT scale.

5 are roughly the same as Figs. 2 and 3. Nonetheless, we can still find the small effects of the one-loop $U(1)_Y$ D -term. In Model (III), the lower bound on the mass for \tilde{u}_3 is tighter than that for \tilde{q}_3 , since a positive one-loop $U(1)_Y$ D -term is generated. In Model (IV), the lower bound on the mass for \tilde{e}_3 is much larger than that in Model (II) because of a negative one-loop $U(1)_Y$ D -term.

4 Constraints from K^0 - \bar{K}^0 Mixing

K^0 - \bar{K}^0 mixing gives the severest constraints on the masses for the first-two generation sfermions in flavor-changing processes. In this section, we calculate the lower bounds on the heavy-sfermion mass scale m_D from the K_L - K_S mass difference Δm_K and the CP-violating parameter ϵ_K . Combining these bounds on m_D with the bounds on the mass ratio m_0/m_D given in the previous section, we obtain the lower bounds on the light sfermion masses m_0 at the GUT scale. It turns out that no severe fine tuning is needed in the NESSM, compared with the original Effective SUSY SM (ESSM).

We first discuss the structure of the mass matrix for the down-type squarks, since it induces a dominant flavor violation in K^0 - \bar{K}^0 mixing through the gluino-squark box diagram. We restrict our attention to the contribution from the heavy first-two generation squarks below. The mass matrix for the down-type squarks in the $U(1)_X$ gauge basis is given by

$$M^2 = \begin{pmatrix} M_{LL}^2 & \mathbf{0} \\ \mathbf{0} & M_{RR}^2 \end{pmatrix}, \quad (25)$$

where we have set the left-right mixing mass terms zero due to the smallness of the corresponding Yukawa couplings. The left- and right-handed squark mass matrices M_{LL}^2 and M_{RR}^2 consist

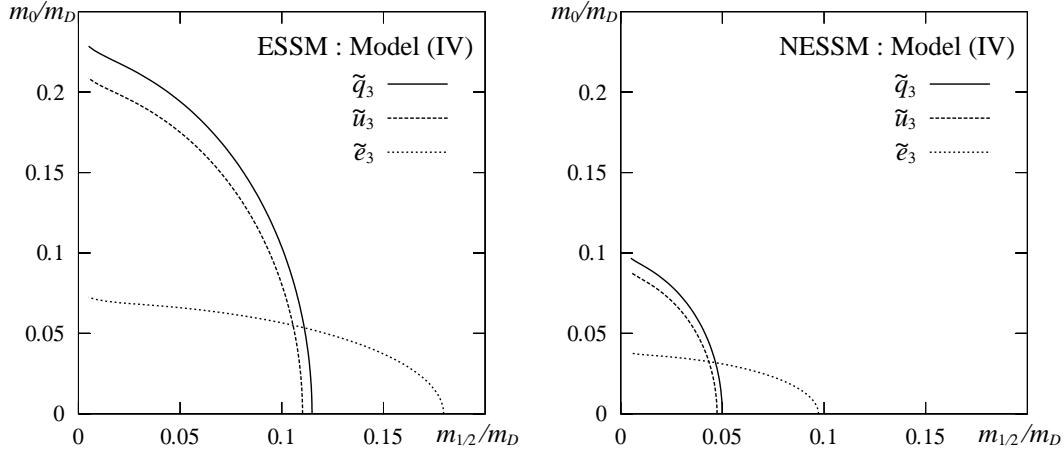


Figure 5: Lower bounds on the mass ratios of the light sfermions to the heavy sfermions in Model (IV). We have set $m_D = 10$ TeV and $m_\psi = 2.7 \times m_D$ at the GUT scale.

of two parts as follows:

$$M_{LL}^2 = M_{LL,D}^2 + M_{LL,0}^2, \quad M_{RR}^2 = M_{RR,D}^2 + M_{RR,0}^2. \quad (26)$$

$M_{LL,D}^2$ and $M_{RR,D}^2$ come from the $U(1)_X$ D -term and are proportional to their $U(1)_X$ charges,

$$M_{LL,D}^2 = \begin{pmatrix} Q_{\mathbf{10}_1}^X & 0 \\ 0 & Q_{\mathbf{10}_2}^X \end{pmatrix} m_D^2, \quad M_{RR,D}^2 = \begin{pmatrix} Q_{\mathbf{5}^{*1}}^X & 0 \\ 0 & Q_{\mathbf{5}^{*2}}^X \end{pmatrix} m_D^2. \quad (27)$$

$M_{LL,0}^2$ and $M_{RR,0}^2$ are contributions from supergravity, whose components are of the order of the light-sfermion mass squared m_0^2 and induce flavor violation,⁴

$$M_{LL,0}^2 = \begin{pmatrix} O(m_0^2) & O(m_0^2) \\ O(m_0^2) & O(m_0^2) \end{pmatrix}, \quad M_{RR,0}^2 = \begin{pmatrix} O(m_0^2) & O(m_0^2) \\ O(m_0^2) & O(m_0^2) \end{pmatrix}. \quad (28)$$

The SUSY contribution to K^0 - \bar{K}^0 mixing is controlled by two parameters qualitatively, as shown in Appendix C: the averaged squark masses $\bar{m}_{LL, RR}^2$ and the off-diagonal elements $\delta_{LL, RR}$.⁵ The averaged squark masses \bar{m}_{LL}^2 and \bar{m}_{RR}^2 are given by

$$\bar{m}_{LL}^2 = \sqrt{Q_{\mathbf{10}_1}^X Q_{\mathbf{10}_2}^X} m_D^2, \quad \bar{m}_{RR}^2 = \sqrt{Q_{\mathbf{5}^{*1}}^X Q_{\mathbf{5}^{*2}}^X} m_D^2, \quad (29)$$

where we have neglected $O(m_0^2)$ contributions. On the other hand, δ_{LL} and δ_{RR} are the off-diagonal elements of the squark mass matrices normalized with the averaged squark masses, in

⁴ If the $U(1)_X$ charges for the first and second generations are different, the off-diagonal elements of $M_{LL,0}^2$ and $M_{RR,0}^2$ are suppressed by $(\langle \Phi \rangle / M_{\text{Pl}})^{|Q_{\mathbf{10}_1}^X - Q_{\mathbf{10}_2}^X|}$ and $(\langle \Phi \rangle / M_{\text{Pl}})^{|Q_{\mathbf{5}^{*1}}^X - Q_{\mathbf{5}^{*2}}^X|}$, respectively.

⁵In the following figures, we use the exact formula for the K^0 - \bar{K}^0 mixing given in Appendix C.

a basis where the quark mass matrix is diagonal (see also Appendix C). In order to change the $U(1)_X$ gauge basis to this basis, we introduce unitary matrices V^L and V^R which diagonalize the down-type quark Yukawa matrix given in the $U(1)_X$ gauge basis. We parameterize them as

$$V^L = \begin{pmatrix} \cos \theta_L & -e^{i\alpha_L} \sin \theta_L \\ e^{i\alpha_L} \sin \theta_L & \cos \theta_L \end{pmatrix}, \quad (30)$$

and V^R with the replacement $L \rightarrow R$ in V^L . Then, the squark mass matrices, \mathcal{M}_{LL}^2 and \mathcal{M}_{RR}^2 , in a basis where the quark mass matrix is diagonal are given by

$$\mathcal{M}_{LL}^2 = V^{L\dagger} M_{LL}^2 V^L, \quad \mathcal{M}_{RR}^2 = V^{R\dagger} M_{RR}^2 V^R. \quad (31)$$

Consequently, the off-diagonal element δ_{LL} is represented as follows:

$$\delta_{LL} = -e^{i\alpha_L} \sin \theta_L \cos \theta_L (Q_{\mathbf{10}_1}^X - Q_{\mathbf{10}_2}^X) \frac{m_D^2}{\bar{m}_{LL}^2} + O\left(\frac{m_0^2}{\bar{m}_{LL}^2}\right), \quad (32)$$

and δ_{RR} is obtained with the replacement $L \rightarrow R$ and $Q_{\mathbf{10}_i}^X \rightarrow Q_{\mathbf{5}^*_i}^X$. If $Q_{\mathbf{10}_1}^X \neq Q_{\mathbf{10}_2}^X$, δ_{LL} is dominated by the $U(1)_X$ D -term contribution and is given definitely up to the mixing angle $\sin \theta_L$. On the other hand, if $Q_{\mathbf{10}_1}^X = Q_{\mathbf{10}_2}^X$, the $U(1)_X$ D -term contribution vanishes and the second term dominates δ_{LL} . In this case, the off-diagonal element has a large model dependence since we cannot calculate the supergravity contributions.

The contribution to K^0 - \bar{K}^0 mixing from the gluino-squark box diagram is calculated in Appendix C. We have included the leading-order QCD corrections [24, 12] and used the bag parameters obtained by lattice calculations [25, 26]. The details are given in Appendix C. The constraints from Δm_K are summarized as follows:

$$\delta_{LL, RR} < \frac{\bar{m}_{LL, RR}}{(25 \sim 35) \text{ TeV}}, \quad (33)$$

$$(\delta_{LL} \delta_{RR})^{1/2} < \frac{(\bar{m}_{LL} \bar{m}_{RR})^{1/2}}{(150 \sim 250) \text{ TeV}}. \quad (34)$$

This shows that if δ_{LL} and δ_{RR} are of the same order, Eq. (34) gives about ten times as severe bounds on the heavy squark masses as Eq. (33). Furthermore, if δ_{LL} and/or δ_{RR} have CP-violating phases of order unity, the constraints from ϵ_K give twelve times severer bounds,

$$\delta_{LL, RR} < \frac{\bar{m}_{LL, RR}}{(310 \sim 430) \text{ TeV}}, \quad (35)$$

$$(\delta_{LL} \delta_{RR})^{1/2} < \frac{(\bar{m}_{LL} \bar{m}_{RR})^{1/2}}{(1900 \sim 3100) \text{ TeV}}, \quad (36)$$

as discussed in Appendix C.

Now, let us discuss Models (I-IV) in turn. In Model (I), the averaged squark masses are

$$\bar{m}_{LL}^2 = \bar{m}_{RR}^2 = m_D^2. \quad (37)$$

Since the $U(1)_X$ charges for the first-two generations are the same, the D -term contribution does not induce any flavor violation. Thus, Model (I) is the hybrid scenario of the decoupling and degeneracy in a sense. Then, the flavor violation comes from the supergravity contributions. Although we cannot calculate the off-diagonal elements in this case, we expect $\delta_{LL,RR}$ to be of order $(0.1 \sim 1)m_0^2/\bar{m}_{LL,RR}$. For simplicity, we here assume

$$\delta_{LL} = \frac{m_0^2}{m_D^2}, \quad \delta_{RR} = \frac{m_0^2}{m_D^2}. \quad (38)$$

Then, we obtain the bound on the light sfermion mass m_0 from Eq. (34) as

$$m_0 > (150 \sim 250) \left(\frac{m_0}{m_D} \right)^3 \text{ TeV}, \quad (39)$$

if δ_{LL} and δ_{RR} do not have CP-violating phases. The ratio m_0/m_D is bounded from below in Fig. 2 in the previous section, so that the lower bounds on m_0 can be estimated as follows:

ESSM	NESSM
$m_0/m_D \geq 0.18$	$m_0/m_D \geq 0.07$
$m_0 > (0.9 \sim 1.5) \text{ TeV}$	$m_0 > (60 \sim 100) \text{ GeV}$

(40)

Here, the bounds on m_0/m_D are those for zero gaugino mass. Note that the lower bound on m_0 depends on the third power of the ratio m_0/m_D in Eq. (39). As a result, although the bound on m_0/m_D in the NESSM is only 1/2.5 of that in the ESSM, the bound on the light sfermion mass m_0 in the NESSM becomes much smaller than in the ESSM by a factor of $(1/2.5)^3 \sim 1/15$.

If δ_{LL} and δ_{RR} have CP phases of order unity, the constraint from ϵ_K gives twelve times larger bounds on m_0 than that from Δm_K as,

ESSM	NESSM
$m_0/m_D \geq 0.18$	$m_0/m_D \geq 0.07$
$m_0 > (11 \sim 18) \text{ TeV}$	$m_0 > (0.8 \sim 1.3) \text{ TeV}$

(41)

Thus, it seems to require some tuning of the phases or electroweak symmetry breaking even in the NESSM. In order to reduce the bound by a factor of 3, for example, the phase of $\delta_{LL}\delta_{RR}$ must be tuned to be $1/3^2 \sim 0.1$. However, including the gaugino contributions reduces the lower bound on m_0 , so that we can, in fact, realize the hierarchical spectrum without tuning as shown in Fig. 6.

In Fig. 6, we have plotted the lower bounds on m_0 , the masses for the various light squarks and sleptons at the GUT scale, including the gaugino contributions. The horizontal axis represents the running gluino mass evaluated at the gluino decoupling scale, which is twice as large as the GUT-scale gaugino mass $m_{1/2}$ in the previous section. The boundary conditions at the GUT scale, such as $m_\psi = 2.0 \times m_D$, are the same as in the previous section. The only difference from the previous section is that m_D is not fixed to 10 TeV but varies with the constraints from $K^0\bar{K}^0$ mixing.

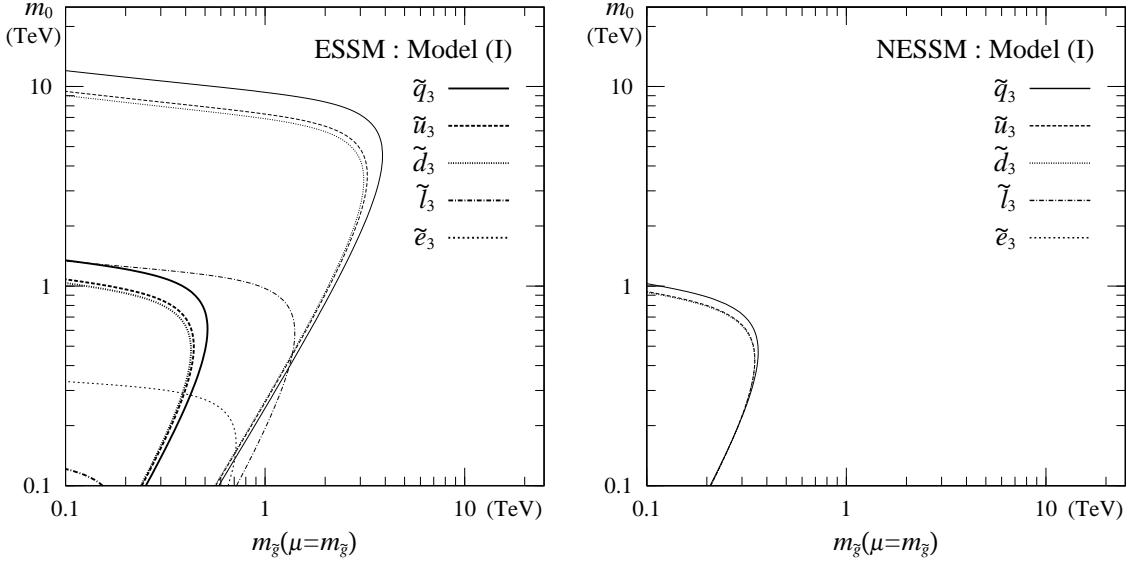


Figure 6: Lower bounds on the bare masses for the light sfermions in Model (I). The horizontal axis represents the gluino running mass evaluated at the gluino decoupling scale, and the vertical axis the indicated light sfermion mass at the GUT scale. The regions below the curves are excluded due to the negative mass squareds for the corresponding sfermions. Bold lines represent the constraints from Δm_K . Fine lines represent the constraints from ϵ_K with $O(1)$ CP phases.

Fig. 6 shows that there is no constraints from Δm_K in the NESSM. Moreover, the NESSM solves the CP problem in K^0 - \bar{K}^0 mixing without tuning of the phases or electroweak symmetry breaking. Note that the shape of the excluded region in Fig. 6 is very different from that in Fig. 2. This is because m_D can take a smaller value as m_0 decreases,

$$m_D > \text{const.} \times (m_0)^{2/3}, \quad (42)$$

as can be seen from Eq. (39). This is peculiar to the case where the flavor violation comes from the supergravity contributions. In contrast, in the case where the $U(1)_X$ D -term contributes to the flavor violation such as in Models (II-IV), the lower bound on m_D does not change with m_0 , so that the shape of the excluded region of m_0 is similar to that of m_0/m_D in the previous section.

Now, we turn to the other models. In Model (II), the averaged squark masses are given by

$$\bar{m}_{LL}^2 = \sqrt{2}m_D^2, \quad \bar{m}_{RR}^2 = m_D^2. \quad (43)$$

The left-handed down-type squarks of the first and second generations have different $U(1)_X$ charges, so that the $U(1)_X$ D -term contribution induces flavor violation. In this case, the mixing angle θ_L is necessary to determine δ_{LL} . Since the product of V^L and the diagonalizing

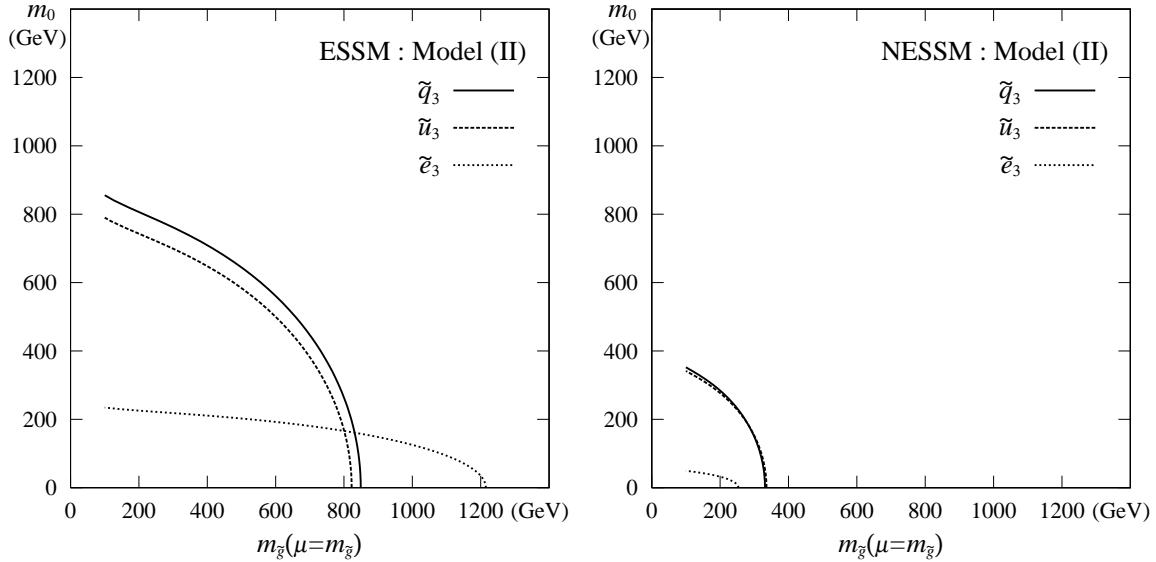


Figure 7: Lower bounds on the bare masses for the light sfermions in Model (II). The constraints from Δm_K (Eq. (33)) are plotted.

matrix for the left-handed up-type quark gives the Cabibbo angle $\sin \theta_C = 0.22$, it is natural to take $\sin \theta_L \sim \sin \theta_C$. Thus, we here take $\sin \theta_L = 0.22$ and set the off-diagonal element δ_{LL} as

$$\delta_{LL} = \frac{\sin \theta_L \cos \theta_L}{\sqrt{2}} \sim 0.15. \quad (44)$$

On the other hand, δ_{RR} is determined from the supergravity contributions and is more model-dependent than δ_{LL} . Therefore, we here restrict our attention to the flavor violation from the $U(1)_X$ D -term contribution and give the lower bounds on the light sfermion masses using only the constraint on δ_{LL} . The supergravity contributions are discussed later.

Then, we obtain the following bound from Eq. (33):

$$m_0 > (3.2 \sim 4.4) \left(\frac{m_0}{m_D} \right) \text{ TeV}. \quad (45)$$

Since the bounds on the ratio m_0/m_D are given in Fig. 3 in the previous section, we can estimate the lower bounds on m_0 from Δm_K as

ESSM	NESSM
$m_0/m_D \geq 0.22$	$m_0/m_D \geq 0.09$
$m_0 > (700 \sim 970) \text{ GeV}$	$m_0 > (290 \sim 400) \text{ GeV}$

(46)

In Fig. 7, we have plotted the lower bounds on m_0 in Model (II) including the effect of the gaugino masses. It shows that we can take more natural mass scale for the light sfermions in the NESSM than in the ESSM. If δ_{LL} has a phase of order unity, however, the lower bounds on

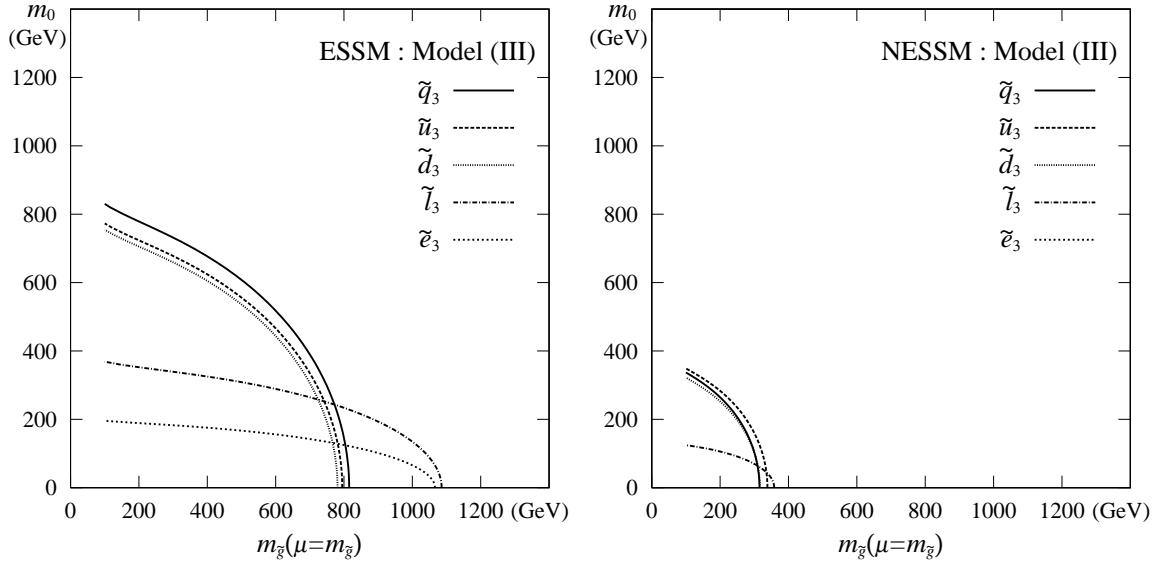


Figure 8: Lower bounds on the bare masses for the light sfermions in Model (III). The constraints from Δm_K (Eq. (33)) are plotted.

the light sfermion masses become higher by a factor of 12. Therefore, the phase is necessary to be less than 10^{-2} to avoid extreme fine tuning of the electroweak symmetry breaking.

Before turning to Model (III), we comment on the supergravity contributions. If we take $\delta_{RR} = m_0^2/m_D^2$ as we did in Model (I), we obtain the following lower bounds on m_0 from Eq. (34): $m_0 > (2.6 \sim 4.3)$ TeV in the ESSM and $m_0 > (430 \sim 720)$ GeV in the NESSM. Thus, it seems to require a slight tuning of the electroweak symmetry breaking even in the NESSM. However, the size of δ_{RR} is strongly model-dependent as we emphasized, and if we take $\delta_{RR} = (1/4) \times m_0^2/m_D^2$, for example, the bounds on m_0 become half. Moreover, the bounds are greatly lowered by including the effect of the gaugino masses as in Fig. 6. Therefore, the supergravity contributions are less important to restrict the parameters in Model (II).

In Fig. 8, we have plotted the lower bounds on m_0 in Model (III). Here, we have taken $\sin \theta_L = 0.22$ as in Model (II). Since the $U(1)_X$ charge assignment for the first-two generations in Model (III) is the same as in Models (II), $K^0-\bar{K}^0$ mixing gives the same constraints on both models. The lower bounds on the mass ratios m_0/m_D are also almost the same between Models (II) and (III) as shown in the previous section. Therefore, the bounds on m_0 in Fig. 8 are similar to those in Fig. 7.

In Model (IV), both δ_{LL} and δ_{RR} come from the large $U(1)_X$ D -term contribution, so that the constraint is the severest among Models (I-IV). The averaged squark masses are given by

$$\bar{m}_{LL}^2 = \sqrt{2}m_D^2, \quad \bar{m}_{RR}^2 = \sqrt{2}m_D^2. \quad (47)$$

If we set both angles θ_L and θ_R equal to the Cabibbo angle for simplicity, then the δ_{LL} and δ_{RR}

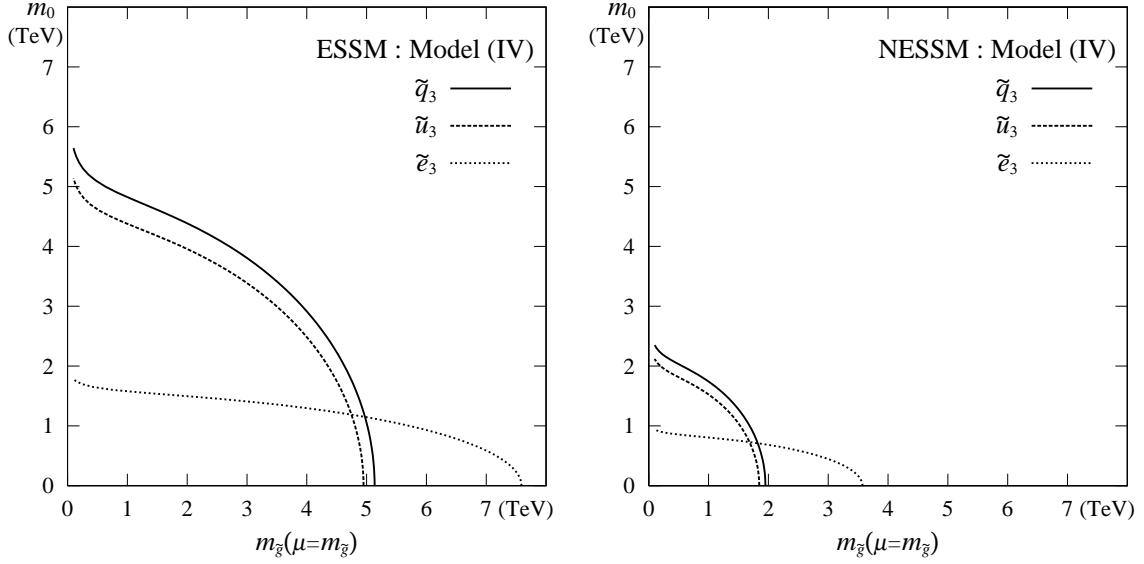


Figure 9: Lower bounds on the bare masses for the light sfermions in Model (IV). The constraints from Δm_K (Eq. (34)) are plotted.

are given by

$$\delta_{LL} = \frac{\sin \theta_L \cos \theta_L}{\sqrt{2}} \sim 0.15, \quad \delta_{RR} = \frac{\sin \theta_R \cos \theta_R}{\sqrt{2}} \sim 0.15. \quad (48)$$

Substituting these values into Eq. (34), we obtain the following bound:

$$m_0 > (19 \sim 32) \left(\frac{m_0}{m_D} \right) \text{ TeV}. \quad (49)$$

The bounds on the ratio m_0/m_D are given in Fig. 5, so that the lower bounds on m_0 can be estimated as

ESSM	NESSM
$m_0/m_D \geq 0.23$	$m_0/m_D \geq 0.10$
$m_0 > (4.4 \sim 7.4) \text{ TeV}$	$m_0 > (1.9 \sim 3.2) \text{ TeV}$

(50)

In Fig 9, we have plotted the lower bounds on m_0 in Model (IV) including the effect of the gaugino mass. It shows that an extreme fine tuning is required in Model (IV). We can reduce the degree of fine tuning by taking smaller values for the mixing angles $\sin \theta_L$ and $\sin \theta_R$. However, in order to reduce the bounds on the masses for both the light sfermions and the gluino to 500 GeV, we must take $\sin \theta_R$ less than 0.03 and it causes another tuning problem. In consequence, it seems difficult to realize Model (IV) without fine tuning in a framework of the NESSM, even if there is no CP phase.

5 Other Renormalization-Group Effects

In this section, we discuss other RG effects on the light sfermion masses which we have not considered in the previous sections. There are three types of contributions which potentially affect the previous analyses: the three-loop contribution through the gauge couplings, the contribution from the bottom and tau Yukawa couplings, and the contribution from the top Yukawa coupling.

First, we consider the correction to the light sfermions from the RG equations at the three-loop level. We here adopt the $\overline{\text{SDR}}$ scheme defined by “the analytic continuation into superspace” [27]. The RG equations for the SUSY-breaking scalar mass squareds in this scheme coincide with those in the $\overline{\text{DR}'}$ scheme at the two-loop level, so that $\overline{\text{SDR}}$ scheme is considered to be all-order definition of the $\overline{\text{DR}'}$ scheme.

When the SUSY-breaking scalar mass squareds are regarded as a D -term of an external $U(1)_X$ gauge multiplet, the RG contributions from the gauge interactions can be divided into two classes in the limit of vanishing gaugino mass in this scheme. One is the contribution from mixed anomalies between the external $U(1)_X$ and the internal gauge symmetries. The other is the contribution from the kinetic-term mixing between $U(1)$ symmetries, which exists only if there is an internal $U(1)$ gauge symmetry. Since we have imposed Eqs. (3, 4, 5) in the NESSM, the RG contributions to the light sfermion masses from the heavy sfermions and extra matters only come from the $U(1)_Y$ and $U(1)_X$ mixing contribution at the three-loop level [15, 28]. Then, the contributions are suppressed by small α_Y and negligible compared with the two-loop finite correction. Furthermore, in models where the extra matters are the fundamental representation of $SU(3)_C$ and $SU(2)_L$, the contributions from the heavy sfermions and extra matters in the three-loop RG equations are proportional to $\alpha_3\alpha_2\alpha_Y$ or α_Y^3 , since the terms proportional to $\alpha_3^2\alpha_Y$ and $\alpha_2^2\alpha_Y$ vanish due to Eqs. (4, 5) in this case. Thus, the three-loop RG contributions are more strongly suppressed in such models.

Next, we consider the effect of the bottom and tau Yukawa couplings. In Models (II, IV) the right-handed bottom squark and the left-handed slepton of the third generation obtain masses from the $U(1)_X$ D -term due to $Q_{5*3}^X \neq 0$. Then, the bottom and tau Yukawa couplings may drive the mass squareds for the doublet squark of the third generation and the right-handed stau to be negative at the one-loop level. The RG contributions to the doublet squark of the third generation and the right-handed stau, $m_{\tilde{q}_3, \text{yukawa}}^2$ and $m_{\tilde{e}_3, \text{yukawa}}^2$, are given as

$$m_{\tilde{q}_3, \text{yukawa}}^2 \simeq -0.003\% \times (1 + \tan^2 \beta)(m_D^2 + |A_b|^2), \quad (51)$$

$$m_{\tilde{e}_3, \text{yukawa}}^2 \simeq -0.005\% \times (1 + \tan^2 \beta)(m_D^2 + |A_\tau|^2). \quad (52)$$

Here, we have assumed that $\tan \beta$ is not so large, and A_b and A_τ are the soft SUSY-breaking trilinear scalar couplings associated with the bottom and tau Yukawa couplings. Then, the RG contribution to the doublet quark of the third generation is negligible if $\tan \beta < 10$, compared with the finite correction. On the other hand, the RG contribution to the right-handed stau can be larger than the finite correction. However, the bound on the bare mass given by these contributions is still looser than the bounds on the light squarks from the finite correction, as long as $\tan \beta$ is sufficiently small. Since Models (II, IV) predict the small bottom and tau

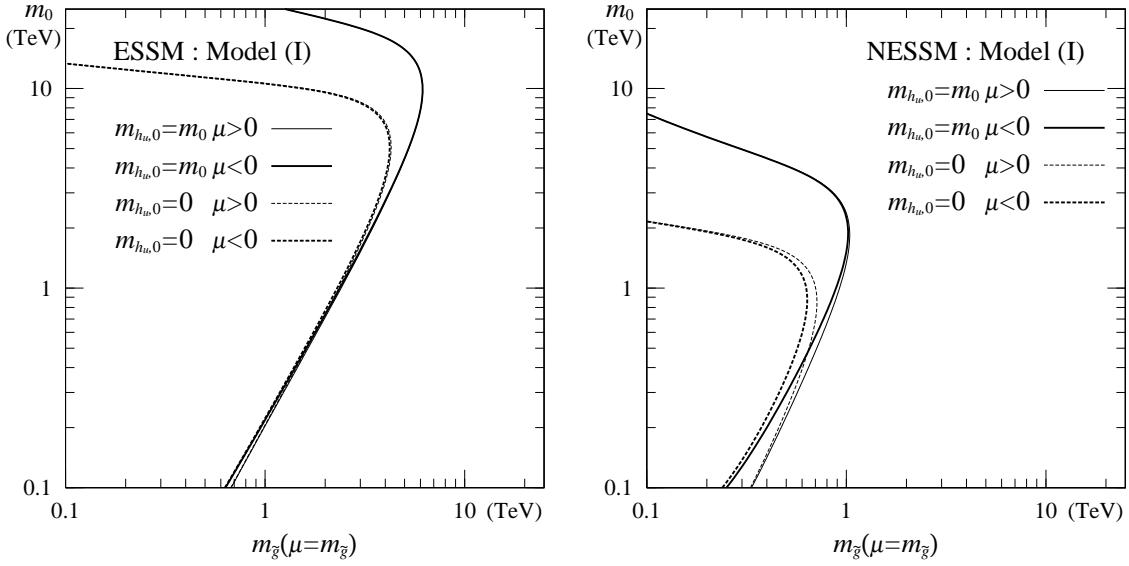


Figure 10: Lower bounds on the bare masses for the light sfermions in Model (I) are plotted by combining the constraint from ϵ_K (Eq. (36)) with the condition that $m_{\tilde{t}_1}, m_{\tilde{t}_2} > 0$, where \tilde{t}_1 and \tilde{t}_2 represent the mass eigenstates for top squarks. The regions below the curves are excluded due to the negative mass-squared eigenvalues for top squarks. Here, we have set $\tan \beta = 3.0$.

Yukawa coupling constants due to $Q_{5^*}^X \neq 0$ and thus require small $\tan \beta$, these contributions are sufficiently suppressed.

Finally, we discuss the effect of the top Yukawa coupling. In the previous sections, we have neglected it, since the effect of the top Yukawa coupling depends on the supergravity contributions, which are model-dependent. There are two negative contributions to the top-squark mass squared through the top Yukawa coupling. One is the RG contribution which mainly comes from the up-type Higgs mass m_{h_u} . The other is the contribution from the left-right mixing term, which gives the negative contribution in diagonalizing the top-squark mass matrix. The left-right mixing term for top squark is given by

$$(M_{\tilde{t}}^2)_{LR} = m_t(A_t + \mu \cot \beta), \quad (53)$$

where m_t is the top quark mass, A_t the soft SUSY-breaking trilinear coupling associated with the top Yukawa coupling and μ the supersymmetric mass for the Higgs doublets. Thus, it turns out that μ and A_t play an important role in calculating the negative contributions. However, A_t at the weak scale is almost saturated by the radiative correction from the gluino, so that we take $A_t = 0$ at the GUT scale for simplicity in the following calculations. The size of μ is determined by the electroweak symmetry breaking.

In Figs. 10 and 11, we have plotted the lower bounds on the bare masses for top squarks. Here, we have assumed the universal scalar mass at the GUT scale except for the up-type Higgs mass, and taken $A_t = 0$ at the GUT scale for simplicity. In the figures, we have shown two extreme cases for the up-type Higgs mass in each choice for the sign of the μ parameter: one is

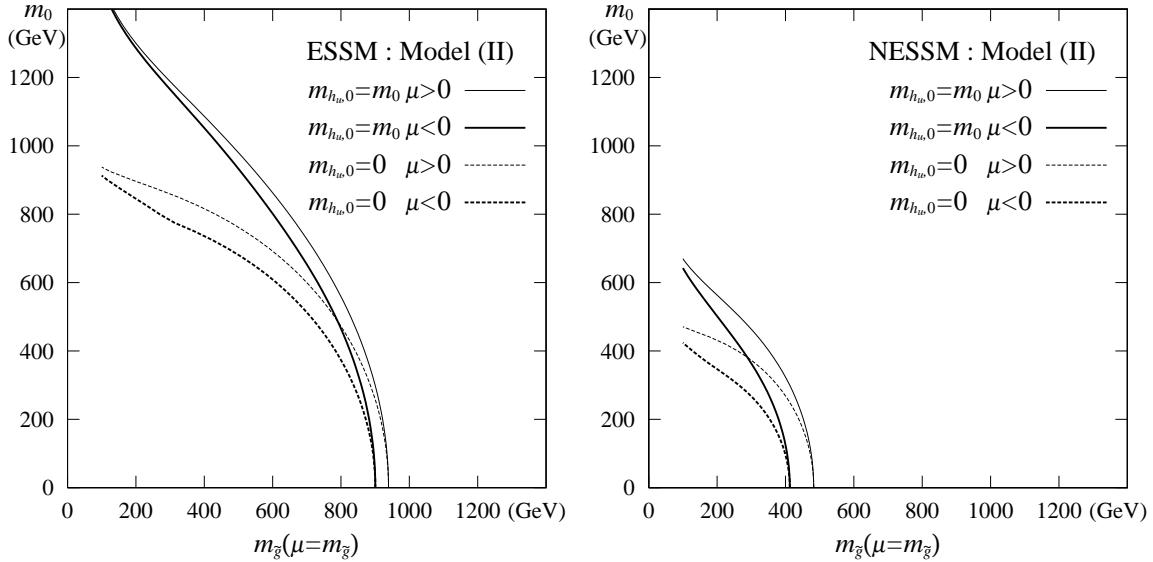


Figure 11: Lower bounds on the bare masses for the light sfermions in Model (II) are plotted by combining the constraint from Δm_K (Eq. (33)) with the condition that $m_{t_1}^2, m_{t_2}^2 > 0$. The regions below the curves are excluded due to the negative mass-squared eigenvalues for top squarks. Here, we have set $\tan \beta = 3.0$.

the universal case $m_{h_u} = m_0$ and the other is $m_{h_u} = 0$. If μ is positive, the A_t and μ are added up constructively in the left-right mixing term, since A_t receive the positive contribution from the gaugino mass. Thus, the lower bound on the top squark mass is somewhat severe in this case. On the other hand, if μ is negative, the cancellation between two contributions from A_t and μ occurs, so that the bound is weaker than the case with $\mu > 0$.

In the region where m_0 is much larger than the gaugino masses, the up-type Higgs mass m_{h_u} gives the dominant effect. Thus, the effect of the top Yukawa coupling is much model-dependent in this region, since we cannot predict the up-type Higgs mass at the GUT scale. For instance, if the up-type Higgs mass is much smaller than the top squark mass at the GUT scale, the effect is negligible as shown in Figs. 10 and 11. On the other hand, in the region where m_0 is small enough, the dominant effect of the top Yukawa coupling comes from the left-right mixing term through diagonalization of the top-squark mass matrix. Since A_t term is generated radiatively by the gluino as $A_t \sim m_{\tilde{g}}$, it gives the negative contribution to the top-squark mass eigenvalue of order $-m_t A_t \sim -m_t m_{\tilde{g}}$. Thus, in order to obtain the positive top-squark mass squared at the weak scale, we have to take a larger mass for the gluino than the case without top-Yukawa contributions. The required increase $\delta m_{\tilde{g}}$ of the gluino mass is determined by the following inequality: $2m_{\tilde{g}} \delta m_{\tilde{g}} \gtrsim m_t m_{\tilde{g}}$, since the positive contribution from the gluino is estimated as of order $m_{\tilde{g}}^2$. That is, we have to take a larger mass for the gluino by the order of 100 GeV compared with the previous case as shown in Figs. 10 and 11.

To summarize, the lower bounds on the light sfermion masses become somewhat severer by including the effects of the top Yukawa coupling. However, we have found that we can still

take the light sfermion and gluino masses as small as ~ 400 GeV in the natural effective SUSY, while we have to take them larger than ~ 1 TeV in the original effective SUSY.

6 Conclusions and Discussion

In this article, we have calculated the finite corrections to the light sfermion masses from the heavy sfermions and extra matters at the two-loop level in the NESSM, assuming that all fields are embedded in the $SU(5)_{\text{GUT}}$ multiplets at the GUT scale. While the sfermion mass squareds in the NESSM are RG invariant at the two-loop level in the limit of vanishing gaugino-mass and Yukawa-coupling contributions, the finite corrections may still drive the light-sfermion mass squareds to be negative. The finite corrections increase (decrease) if the supersymmetric masses (the holomorphic SUSY-breaking masses) of the extra matters become large. We have found that the corrections can be less than $(0.3 - 1)\%$ of the heavy-squark mass squareds.

We have also discussed whether the models in which the NESSM is realized by the $U(1)_X$ gauge interaction at the GUT scale are viable or not in the light of the experimental constraints from Δm_K and ϵ_K . On this setup, the supergravity contributions to the SUSY-breaking terms, which are generically non-universal in flavor space, are suppressed compared with the $U(1)_X$ D -term contribution. We have found that, when both the left- and right-handed down-type squarks of the first-two generations have common $U(1)_X$ charges, the supersymmetric contributions to Δm_K and ϵ_K are sufficiently suppressed without breaking naturalness, even if the flavor-violating supergravity contributions to the sfermion mass matrices are included. On the other hand, when only the right-handed squarks of the first and second generations have a common $U(1)_X$ charge, we can still satisfy the constraint from Δm_K naturally, but evading the bound from ϵ_K requires somewhat small CP phase of order 10^{-2} .

The formulae we have given in Appendix B are applicable to any models which have hierarchical mass spectrum for the sfermions such as effective SUSY. In particular, the formulae for the finite corrections can also be applied to the models where the $U(1)_X$ D -term is generated at much lower energy than the GUT scale [8], since they do not have explicit renormalization-point dependence. Then, it gives the lower bounds on the light-sfermion bare masses similar to the ones we have derived in this article. Thus, we believe that our result gives the most conservative bound for the naturalness in the effective SUSY models.

Acknowledgments

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A Anomalous U(1) Model

In this appendix, we briefly review the model given in Ref. [15] and show that it naturally realizes the setup of the NESSM. The model is based on the anomalous U(1) SUSY-breaking model [6].

The anomalous U(1) gauge symmetry frequently appears in low-energy effective theories of string theory. The matter content is anomalous under this U(1) symmetry, but its anomalies are canceled by a nonlinear transformation of the dilaton chiral multiplet [29]. This leads to the generation of a nonzero Fayet-Iliopoulos (FI) D -term ξ which is smaller than the reduced Planck scale M_{pl} by a one-loop factor, $\xi^2 \simeq (g^2/192\pi^2)\text{Tr}QM_{\text{pl}}^2$. We parameterize it as $\xi \equiv \epsilon M_{\text{pl}}$ with $\epsilon = O(0.1)$, and take the sign convention of U(1) charges such that $\xi^2 > 0$.

We identify this anomalous U(1) gauge symmetry with $U(1)_X$ given in the text. Thus, we assign the $U(1)_X$ charges $Q_{\mathbf{10}_i}^X$ and $Q_{\mathbf{5}^*_i}^X$ to the quark and lepton chiral multiplets $\Psi_{\mathbf{10}_i}$ and $\Psi_{\mathbf{5}^*_i}$, respectively ($Q_{\mathbf{10}_i}^X, Q_{\mathbf{5}^*_i}^X \geq 0$). We also introduce extra-matter chiral multiplets $\Psi_{\mathbf{5}_{\text{ex}}}$ and $\Psi_{\mathbf{5}^*_{\text{ex}}}$ with $U(1)_X$ charges $Q_{\mathbf{5}_{\text{ex}}}^X$ and $Q_{\mathbf{5}^*_{\text{ex}}}^X$ ($Q_{\mathbf{5}_{\text{ex}}}^X, Q_{\mathbf{5}^*_{\text{ex}}}^X < 0$), in order to satisfy Eqs. (6, 8). Then, the SUSY-breaking model is constructed as follows.

We consider the $SU(N_c)$ SUSY gauge theory with N_f flavors \mathcal{Q}^a and $\bar{\mathcal{Q}}_{\bar{a}}$ ($N_c/2 < N_f < N_c$), and introduce two singlet chiral superfields Φ_1 and Φ_2 . Here, $a, \bar{a} = 1, \dots, N_f$ represent flavor indices. We assign the $U(1)_X$ charges as $\mathcal{Q}^a(-Q_{\mathbf{5}_{\text{ex}}}^X + Q_{\mathbf{5}^*_{\text{ex}}}^X)/2$, $\bar{\mathcal{Q}}_{\bar{a}}(-Q_{\mathbf{5}_{\text{ex}}}^X + Q_{\mathbf{5}^*_{\text{ex}}}^X)/2$, $\Phi_1(-1)$ and $\Phi_2(1 + Q_{\mathbf{5}_{\text{ex}}}^X + Q_{\mathbf{5}^*_{\text{ex}}}^X)$. The tree-level superpotential of the model is given by

$$W_{\text{tree}} = \frac{\mathcal{Q}^a \bar{\mathcal{Q}}_{\bar{a}}}{M_{\text{pl}}} ((f_\phi)_a^{\bar{a}} \Phi_1 \Phi_2 + (f_\psi)_a^{\bar{a}} \Psi_{\mathbf{5}_{\text{ex}}} \Psi_{\mathbf{5}^*_{\text{ex}}}). \quad (54)$$

Then, the dynamical superpotential is generated by nonperturbative effects of the $SU(N_c)$ gauge interaction, and the effective superpotential of the model is exactly given by

$$\begin{aligned} W_{\text{eff}} = & \frac{1}{M_{\text{pl}}} (\Phi_1 \Phi_2 \text{Tr}(f_\phi M) + \Psi_{\mathbf{5}_{\text{ex}}} \Psi_{\mathbf{5}^*_{\text{ex}}} \text{Tr}(f_\psi M)) \\ & + (N_c - N_f) \left(\frac{\Lambda^{3N_c - N_f}}{\det M} \right)^{\frac{1}{N_c - N_f}}, \end{aligned} \quad (55)$$

in terms of gauge-invariant composite fields $M_{\bar{a}}^a = \mathcal{Q}^a \bar{\mathcal{Q}}_{\bar{a}}$ [30]. Here, Λ is the dynamical scale of the $SU(N_c)$ gauge theory. The D -term potential for $U(1)_X$ is given by

$$\begin{aligned} V_D = & \frac{g_X^2}{2} \left(\xi^2 - (Q_{\mathbf{5}_{\text{ex}}}^X + Q_{\mathbf{5}^*_{\text{ex}}}^X) \text{Tr} \sqrt{M^\dagger M} \right. \\ & \left. - |\Phi_1|^2 + (1 + Q_{\mathbf{5}_{\text{ex}}}^X + Q_{\mathbf{5}^*_{\text{ex}}}^X) |\Phi_2|^2 + Q_{\mathbf{5}_{\text{ex}}}^X |\Psi_{\mathbf{5}_{\text{ex}}}|^2 + Q_{\mathbf{5}^*_{\text{ex}}}^X |\Psi_{\mathbf{5}^*_{\text{ex}}}|^2 \right)^2. \end{aligned} \quad (56)$$

In this appendix, we use the same letter for the chiral superfield and its scalar component.

The dynamics of the model can be understood as follows. First, nonperturbatively generated superpotential forces M to have VEV's, which gives supersymmetric mass terms for the singlet fields and the extra matter fields. The large FI D -term, ξ , is absorbed by the shift of the

singlet fields Φ_1 and Φ_2 . The point is that if f_ψ is larger than f_ϕ , only the singlet fields shift to absorb the FI D -term and the extra-matter fields do not develop VEV's, avoiding phenomenologically disastrous large breaking of the SM gauge groups. We assume that this condition, $f_\psi > f_\phi$, is satisfied. Then, since the singlet fields have the supersymmetric mass induced by the VEV's of M , they cannot absorb ξ completely and nonzero $U(1)_X$ D -term remains. This nonvanishing D -term gives the MSSM sfermions and extra scalars the SUSY-breaking mass squareds proportional to their $U(1)_X$ charges.

Now, let us minimize the potential explicitly. The minimum of the potential can be obtained by making an expansion in the small parameter $(\Lambda/M_{\text{pl}})^{(3N_c-N_f)/N_c} \ll 1$. To the leading order, the VEV's of the fields Φ_1 , Φ_2 and M at the minimum are given by solving the following equations:

$$|\Phi_1|^2 - (1 + Q_{\mathbf{5}_{\text{ex}}}^X + Q_{\mathbf{5}_{\text{ex}}^*}^X) |\Phi_2|^2 = \xi^2, \quad (57)$$

$$\begin{aligned} |\Phi_1|^2 ((N_f - N_c)|\Phi_1|^2 + N_f|\Phi_2|^2) \\ = -(1 + Q_{\mathbf{5}_{\text{ex}}}^X + Q_{\mathbf{5}_{\text{ex}}^*}^X) |\Phi_2|^2 ((N_f - N_c)|\Phi_2|^2 + N_f|\Phi_1|^2), \end{aligned} \quad (58)$$

$$(M^{-1})_a^{\bar{a}} \left(\frac{\Lambda^{3N_c-N_f}}{\det M} \right)^{\frac{1}{N_c-N_f}} = \frac{(f_\phi)_a^{\bar{a}}}{M_{\text{pl}}} \Phi_1 \Phi_2. \quad (59)$$

From Eqs. (57, 58), we find that both the Φ_1 and Φ_2 fields have nonvanishing VEV's of order ξ , and the VEV of the M field is given by Eq. (59) as

$$\langle \Phi_1 \rangle \simeq \langle \Phi_2 \rangle \simeq \xi, \quad \langle M \rangle \simeq \Lambda^{\frac{3N_c-N_f}{N_c}} \left(\frac{M_{\text{pl}}}{\xi^2} \right)^{\frac{N_c-N_f}{N_c}}. \quad (60)$$

Then, the Yukawa matrices for the quarks and leptons are generated through the VEV of the Φ_1 field suppressed by suitable powers of $\langle \Phi_1 \rangle / M_{\text{pl}} \simeq \xi / M_{\text{pl}} = \epsilon$ (see Eq. (13)).

Calculating higher order in $(\Lambda/M_{\text{pl}})^{(3N_c-N_f)/N_c}$, we find that the VEV's of the auxiliary fields can be written in terms of the VEV's of the scalar fields as

$$-(\bar{F}_{M^\dagger})_a^{\bar{a}} = -\frac{N_c - N_f}{N_c N_f} \frac{1}{M_{\text{pl}}} \frac{|\Phi_1|^2 + |\Phi_2|^2}{\Phi_1^* \Phi_2^*} |\text{Tr}(f_\phi M)|^2 (f_\phi^{*-1})_a^{\bar{a}}, \quad (61)$$

$$-\bar{F}_{\Phi_1^\dagger} = \left(\frac{\Phi_2}{M_{\text{pl}}} \right) \text{Tr}(f_\phi M), \quad (62)$$

$$-\bar{F}_{\Phi_2^\dagger} = \left(\frac{\Phi_1}{M_{\text{pl}}} \right) \text{Tr}(f_\phi M), \quad (63)$$

$$-D_X = \frac{2N_f - N_c}{N_c} \frac{1}{M_{\text{pl}}^2} \frac{|\Phi_1|^2 + |\Phi_2|^2}{|\Phi_1|^2 - (1 + Q_{\mathbf{5}_{\text{ex}}}^X + Q_{\mathbf{5}_{\text{ex}}^*}^X) |\Phi_2|^2} |\text{Tr}(f_\phi M)|^2. \quad (64)$$

Substituting Eq. (60) into Eqs. (61 - 64), we obtain

$$-\bar{F}_{M^\dagger} \simeq \left(\frac{\Lambda^{2(3N_c-N_f)}}{\xi^{4(N_c-N_f)} M_{\text{pl}}^{2N_f-N_c}} \right)^{\frac{1}{N_c}}, \quad (65)$$

$$-\bar{F}_{\Phi_1^\dagger} \simeq -\bar{F}_{\Phi_2^\dagger} \simeq \left(\frac{\Lambda^{3N_c-N_f}}{\xi^{2(N_c-N_f)} M_{\text{pl}}^{N_f}} \right)^{\frac{1}{N_c}} \xi, \quad (66)$$

$$-D_X \simeq \left(\frac{\Lambda^{3N_c-N_f}}{\xi^{2(N_c-N_f)} M_{\text{pl}}^{N_f}} \right)^{\frac{2}{N_c}}. \quad (67)$$

The dynamical scale Λ is determined so that $\sqrt{|-D_X|} \simeq (1 \sim 10)$ TeV to give the heavy sfermions multi-TeV masses. From Eqs. (65, 66, 67), we find the following useful relation:

$$|-D_X| \simeq \left| \frac{-F_{\Phi_1}}{\langle \Phi_1 \rangle} \right|^2 \simeq \left| \frac{-F_{\Phi_2}}{\langle \Phi_2 \rangle} \right|^2 \simeq \left| \frac{-F_M}{\langle M \rangle} \right|^2. \quad (68)$$

This relation ensures that flavor-breaking supergravity contributions are an order of magnitude smaller than the contribution from the $U(1)_X$ D -term, since their sizes are estimated as⁶

$$\left| \frac{-F_{\Phi_1}}{M_{\text{pl}}} \right| \simeq \frac{\xi}{M_{\text{pl}}} \left| \frac{-F_{\Phi_1}}{\langle \Phi_1 \rangle} \right| \simeq \epsilon \sqrt{|-D_X|} \simeq (10^2 \sim 10^3) \text{ GeV}. \quad (69)$$

From Eqs. (60, 67), we also find that $\sqrt{|-D_X|} \simeq \langle M \rangle / M_{\text{pl}}$. Thus, the supersymmetric mass for the extra matters is actually the same order with the SUSY-breaking masses for the heavy sfermions.

The gaugino masses arise from the F -term of the dilaton field [21]. Their sizes can be of the order of the weak scale [22], and then it does not much affect the preceding analysis of the dynamics. The SUSY-breaking trilinear scalar couplings of order $\sqrt{|-D_X|}$ are also generated by the superpotential which generates Yukawa matrices for the quarks and leptons, except for the ones which involve only the light sfermions (see Eqs. (14, 15)).

Finally, we comment on the anomaly. We have identified the $U(1)_X$ in the text with the anomalous $U(1)$ gauge symmetry. Then, Eq. (6) might seem contradicted by the fact that the anomalous $U(1)$ gauge symmetry has mixed anomalies for all the other gauge groups including the SM ones. However, it is not a contradiction. Since $U(1)_X$ is broken down at very high energy scale of order ξ , it does not necessarily mean that the matter content is anomalous below ξ scale. That is, if we introduce fields $\Psi_{5_{\text{anom}}}$ and $\Psi_{5^*_{\text{anom}}}$ of masses of order ξ with the superpotential

$$W \sim \langle \Phi_1 \rangle \Psi_{5_{\text{anom}}} \Psi_{5^*_{\text{anom}}}, \quad (70)$$

we can match the anomalies as required by the anomalous $U(1)$ symmetry, keeping Eqs. (6, 8) satisfied between two scales $\sqrt{|-D_X|}$ and ξ . Then, the large two-loop RG contributions are absent below the ξ scale as we have explained above.⁷

⁶ Since the gravitino mass is of the order of the weak scale, we can induce μ -term (supersymmetric mass term for the Higgs field, $W = \mu H_u H_d$) of the desired size (the weak scale) by introducing a holomorphic term $K = H_u H_d$ in the Kähler potential [31].

⁷ The light-sfermion mass squareds may receive negative RG and finite contributions above and at the

B Finite Corrections

In this appendix, we calculate the contributions to the gaugino and the light sfermion masses arising from loops of the extra-matter multiplets and the heavy sfermions. We use the $\overline{\text{DR}}'$ scheme [14] to regularize the theory, since we have adopted the $\overline{\text{SDR}}$ scheme [27] which is all-order definition of the $\overline{\text{DR}}'$ scheme in the text (see Section 5). In this scheme, the ϵ -scalar mass $m_{\tilde{A}}$ is zero at the tree level and it does not appear in the relation between physical quantities. However, if the supertrace of the matter fields is nonzero, $m_{\tilde{A}}$ receives divergent radiative correction at one loop through loops of the sfermions, so that the counterterm is needed to cancel this divergence. The insertion of this counterterm gives divergent contribution to the sfermion masses at one loop. Thus, we have to carefully treat the ϵ -scalar in order to obtain the two-loop contribution to the light sfermion masses when the supertrace is nonvanishing.

First, we explain our notations. The superpotential of the vector-like extra matters, Ψ_{ex} and $\bar{\Psi}_{\text{ex}}$, is given by

$$W_{\text{ex}} = m_\psi \Psi_{\text{ex}} \bar{\Psi}_{\text{ex}}. \quad (71)$$

We denote the scalar and fermion components of Ψ_{ex} ($\bar{\Psi}_{\text{ex}}$) as $\tilde{\psi}_{\text{ex}}$ and ψ_{ex} ($\tilde{\bar{\psi}}_{\text{ex}}$ and $\bar{\psi}_{\text{ex}}$), respectively. In addition, the extra scalars have the SUSY-breaking mass terms

$$\mathcal{L}_{\text{ex, soft}} = -(F_\psi \tilde{\psi}_{\text{ex}} \tilde{\bar{\psi}}_{\text{ex}} + \text{h.c.}) - \tilde{m}^2 \tilde{\psi}_{\text{ex}}^* \tilde{\psi}_{\text{ex}} - \tilde{\bar{m}}^2 \tilde{\bar{\psi}}_{\text{ex}}^* \tilde{\bar{\psi}}_{\text{ex}}. \quad (72)$$

Then, the mass terms for the extra scalars are written as

$$\mathcal{L}_{\text{ex}} = -(\tilde{\psi}_{\text{ex}}^*, \tilde{\bar{\psi}}_{\text{ex}}) \tilde{M}_{\text{ex}}^2 \begin{pmatrix} \tilde{\psi}_{\text{ex}} \\ \tilde{\bar{\psi}}_{\text{ex}} \end{pmatrix}; \quad (73)$$

$$\tilde{M}_{\text{ex}}^2 = \begin{pmatrix} |m_\psi|^2 + \tilde{m}^2 & F_\psi^* \\ F_\psi & |m_\psi|^2 + \tilde{\bar{m}}^2 \end{pmatrix}. \quad (74)$$

The mass matrix \tilde{M}_{ex}^2 can be diagonalized by the unitary matrix V as

$$\text{diag}(m_1^2, m_2^2) = V \tilde{M}_{\text{ex}}^2 V^\dagger. \quad (75)$$

We parameterize V as

$$V = \begin{pmatrix} \cos \theta & -e^{i\alpha} \sin \theta \\ e^{-i\alpha} \sin \theta & \cos \theta \end{pmatrix}, \quad (76)$$

and define

$$y_1 \equiv \frac{m_1^2}{m_\psi^2}, \quad y_2 \equiv \frac{m_2^2}{m_\psi^2}. \quad (77)$$

decoupling scale of $\Psi_{5_{\text{anom}}}$ and $\Psi_{5^*_{\text{anom}}}$. However, we can make these negative contributions smaller than the supergravity contributions by choosing their group-theoretical factors to be small, since there is no large log-factor in the contributions. We include these contributions in the supergravity contributions when we make phenomenological analyses in the text.

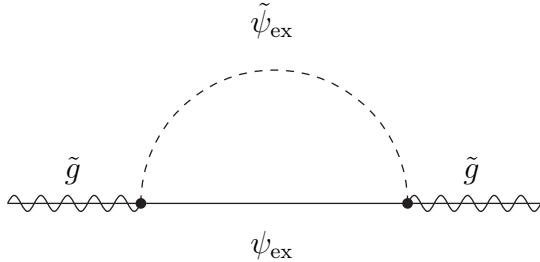


Figure 12: Diagram contributing to the gaugino masses

We calculate the gaugino and the light sfermion masses arising from loops of the extra-matter multiplets in terms of these parameters. Then, the contribution from the heavy sfermions are obtained by taking the limit $m_\psi \rightarrow 0$ and $\theta \rightarrow 0$.

B.1 The gaugino masses

The gauginos acquire their masses through the one-loop diagram of the extra-matter multiplet shown in Fig. 12. If there is a pair of vector-like extra matters, it is given by

$$m_{\tilde{g}_A} = \frac{g_A^2}{4\pi^2} T^A m_\psi \sum_{\alpha=1,2} V_{2\alpha}^\dagger V_{\alpha 1} \frac{m_\alpha^2}{m_\alpha^2 - m_\psi^2} \log \left(\frac{m_\alpha^2}{m_\psi^2} \right), \quad (78)$$

where $A = 1 - 3$ represents the standard-model gauge groups, and we have adopted the SU(5) GUT normalization for the $U(1)_Y$ gauge coupling ($g_1 \equiv \sqrt{5/3} g_Y$). Here, T^A is the Dynkin index of the extra-matter representation, in a normalization $T^A = 1/2$ for a fundamental of $SU(N)$ and $T^1 = (3/5)Y^2$ for $U(1)_Y$. Note that T^A is the Dynkin index of the extra matter and not the sum of the index of the extra matter and anti-matter. That is, we use $T^A = 1/2$ for a vector-like pair of fundamental extra matters.

Then, the gaugino masses are given by

$$m_{\tilde{g}_A} = 2 \left(\frac{\alpha_A}{4\pi} \right) \text{Tr}_R \left[T_R^A m_\psi e^{-i\alpha} \sin 2\theta \frac{y_1 \log y_1 - y_2 \log y_2 - y_1 y_2 \log(y_1/y_2)}{(y_1 - 1)(y_2 - 1)} \right], \quad (79)$$

where the trace is taken over pairs of the extra matters R . (Note that m_ψ, y_1, y_2, α and θ depend on R .) This is in agreement with the result given in Ref. [32].

Obviously, the heavy sfermions do not contribute to the gaugino masses. It can also be seen by taking the limit $m_\psi \rightarrow 0$ and $\theta \rightarrow 0$ in Eq. (79).

B.2 The sfermion masses

We now calculate the light sfermion masses at the two-loop level. There are two types of contributions which generate the light sfermion masses. One is the contribution directly arising

from the loop diagrams: the one-loop graph involving the ϵ -scalar and the two-loop graphs involving heavy particles, both of which are of order α_A^2 . The other contribution is that through the generation of $U(1)_Y$ FI D -term at one loop and two loops, which are of order α_Y and $\alpha_Y\alpha_A$, respectively.

We first consider the direct contribution from the extra-matter multiplets. One diagram contributing to the light sfermion masses is the one-loop ϵ -scalar graph shown in Fig. 13. It contains the counterterm $\delta_{m_{\tilde{A}}^2}$ for the ϵ -scalar mass, $\mathcal{L} = -(1/2)\delta_{m_{\tilde{A}}^2}\tilde{A}_{\mu}^a\tilde{A}_{\mu}^a$, and gives the light-sfermion mass squared,

$$m_{\tilde{f}, \tilde{A}}^2 = \frac{2\epsilon\Gamma(\epsilon)}{(4\pi)^{2-\epsilon}}(\Lambda_{\text{IR}}^2)^{-\epsilon}\mu^{2\epsilon}\sum_A g_A^2 C_{\tilde{f}}^A \delta_{m_{\tilde{A}}^2}, \quad (80)$$

in $D = 4 - 2\epsilon$ dimensions. Here, $C_{\tilde{f}}^A$ is the quadratic Casimir coefficient for the light sfermion \tilde{f} , μ is the renormalization scale, and Λ_{IR} is the infra-red cutoff. The counterterm $\delta_{m_{\tilde{A}}^2}$ is determined to cancel the divergence of the ϵ -scalar mass arising from one loop of the extra-matter multiplets,

$$\text{---} \cdot \text{---} + \text{---} \cdot \text{---} + \text{---} \otimes \text{---} = \text{finite}, \quad (81)$$

as

$$\delta_{m_{\tilde{A}}^2} = -\frac{2g_A^2}{(4\pi)^2} \text{Tr}_R \left[T_R^A (m_1^2 + m_2^2 - 2m_{\psi}^2) \right] \left(\frac{1}{\epsilon} \right). \quad (82)$$

Substituting Eq. (82) into Eq. (80), we obtain

$$m_{\tilde{f}, \tilde{A}}^2 = -4 \sum_A \left(\frac{\alpha_A}{4\pi} \right)^2 C_{\tilde{f}}^A \text{Tr}_R \left[T_R^A m_{\psi}^2 (y_1 + y_2 - 2) \right] \left(\frac{1}{\epsilon} - \log \frac{\Lambda_{\text{IR}}^2}{\mu^2} \right). \quad (83)$$

Here, the combination $-\gamma + \log(4\pi)$ has been absorbed by an appropriate redefinition of the renormalization scale, $\mu \rightarrow \mu\sqrt{e^{\gamma}/4\pi}$ (where γ is the Euler number). The result has $1/\epsilon$ pole of order α_A^2 , so that it contributes to the two-loop RG equations for the light sfermion masses. We will omit μ , hereafter.

The remaining diagram consists of two-loop graphs involving the extra-matter multiplets given in Fig. 14. These graphs are identical to those considered in Ref. [23] in the case of vanishing supertrace. Their contribution is ultra-violet finite even in the case of nonvanishing supertrace. Together with Eq. (83), we obtain the light sfermion masses induced directly by loops of the extra-matter multiplets as

$$m_{\tilde{f}, \text{direct}}^2|_R = -4 \sum_A \left(\frac{\alpha_A}{4\pi} \right)^2 C_{\tilde{f}}^A \text{Tr}_R \left[T_R^A m_{\psi}^2 (y_1 + y_2 - 2) \right] \left(\frac{1}{\epsilon} \right)$$

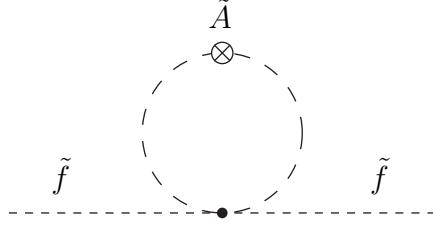


Figure 13: One-loop diagram contributing to the light sfermion masses which involves the ϵ -scalar \tilde{A} .

$$\begin{aligned}
& + 4 \sum_A \left(\frac{\alpha_A}{4\pi} \right)^2 C_{\tilde{f}}^A \text{Tr}_R \left[T_R^A m_\psi^2 \left\{ (y_1 + y_2 - 2)(\log m_\psi^2 + 2) \right. \right. \\
& \quad + (y_1 \log y_1 + y_2 \log y_2) - 2 \left(y_1 \text{Li}_2(1 - \frac{1}{y_1}) + y_2 \text{Li}_2(1 - \frac{1}{y_2}) \right) \\
& \quad \left. \left. + \frac{1}{2} \sin^2 2\theta \left(y_1 \text{Li}_2(1 - \frac{y_2}{y_1}) + y_2 \text{Li}_2(1 - \frac{y_1}{y_2}) \right) \right\} \right], \tag{84}
\end{aligned}$$

which is in agreement with the result given in Ref. [32]. Here, $\text{Li}_2(x) = - \int_0^1 dt \log(1 - xt)/t$ is the dilogarithm function. Note that Λ_{IR}^2 is canceled between the diagrams of Fig. 13 and Fig. 14. Taking the limit $m_\psi \rightarrow 0$ and $\theta \rightarrow 0$ and regarding m_1^2 as the heavy sfermion masses $m_{\tilde{F}}^2$, we obtain the light sfermion masses induced by loops of the heavy sfermions as

$$\begin{aligned}
m_{\tilde{f}, \text{direct}}^2|_{\tilde{F}} &= -4 \sum_A \left(\frac{\alpha_A}{4\pi} \right)^2 C_{\tilde{f}}^A \text{Tr}_{\tilde{F}} \left[T_{\tilde{F}}^A m_{\tilde{F}}^2 \right] \left(\frac{1}{\epsilon} \right) \\
&\quad + 4 \sum_A \left(\frac{\alpha_A}{4\pi} \right)^2 C_{\tilde{f}}^A \text{Tr}_{\tilde{F}} \left[T_{\tilde{F}}^A m_{\tilde{F}}^2 \left(\log m_{\tilde{F}}^2 + 2 - \frac{1}{3}\pi^2 \right) \right], \tag{85}
\end{aligned}$$

where \tilde{F} denotes the heavy sfermions. The direct contribution $m_{\tilde{f}, \text{direct}}^2$ to the light sfermion masses is given by summing up that from the extra-matter multiplets and that from the heavy sfermions,

$$m_{\tilde{f}, \text{direct}}^2 = m_{\tilde{f}, \text{direct}}^2|_R + m_{\tilde{f}, \text{direct}}^2|_{\tilde{F}}. \tag{86}$$

We next consider the light sfermion masses induced via the generation of $U(1)_Y$ FI D -term. At the one-loop level, the light sfermion masses generated by loops of the extra-matter multiplets are given by (see Fig. 15)

$$\begin{aligned}
m_{\tilde{f}, \text{FI-1loop}}^2|_R &= - \left(\frac{\alpha_Y}{4\pi} \right) Y_{\tilde{f}} \text{Tr}_R \left[Y_R \cos 2\theta (m_1^2 - m_2^2) \right] \left(\frac{1}{\epsilon} + 1 \right) \\
&\quad + \left(\frac{\alpha_Y}{4\pi} \right) Y_{\tilde{f}} \text{Tr}_R \left[Y_R \cos 2\theta \left(m_1^2 \log m_1^2 - m_2^2 \log m_2^2 \right) \right]. \tag{87}
\end{aligned}$$

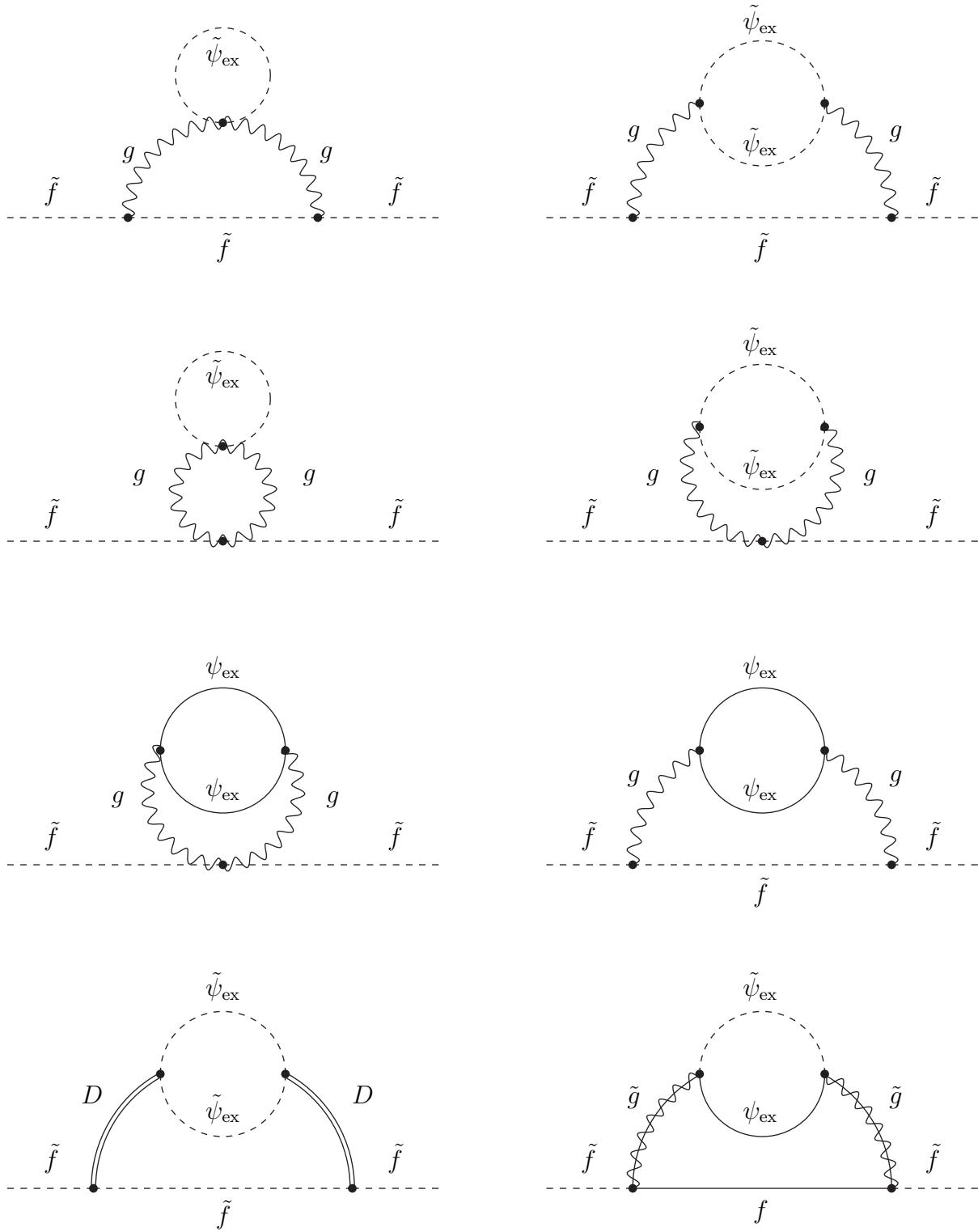


Figure 14: Two-loop diagram contributing to the light sfermion masses which involves the extra-matter multiplets Ψ_{ex} and $\bar{\Psi}_{\text{ex}}$.

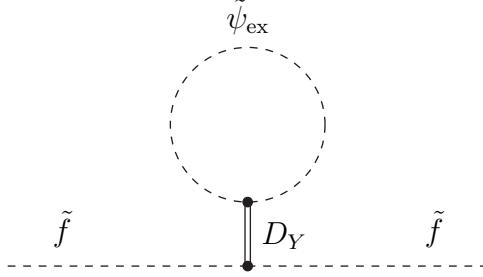


Figure 15: One-loop diagram contributing to the light sfermion masses through the generation of $U(1)_Y$ FI D -term.

This contribution vanishes if the extra-matter multiplets have an invariance under the parity $\Psi_{\text{ex}} \leftrightarrow \bar{\Psi}_{\text{ex}}$, as can be readily seen by taking $\theta = \pm\pi/4$ in the expression. The heavy sfermions also generate one-loop $U(1)_Y$ FI D -term, which gives

$$\begin{aligned} m_{\tilde{f}, \text{FI-1loop}}^2|_{\tilde{F}} &= -\left(\frac{\alpha_Y}{4\pi}\right) Y_{\tilde{f}} \text{Tr}_{\tilde{F}} \left[Y_{\tilde{F}} m_{\tilde{F}}^2 \right] \left(\frac{1}{\epsilon} + 1 \right) \\ &\quad + \left(\frac{\alpha_Y}{4\pi}\right) Y_{\tilde{f}} \text{Tr}_{\tilde{F}} \left[Y_{\tilde{F}} m_{\tilde{F}}^2 \log m_{\tilde{F}}^2 \right]. \end{aligned} \quad (88)$$

The total contribution through the generation of $U(1)_Y$ FI D -term at one loop is

$$m_{\tilde{f}, \text{FI-1loop}}^2 = m_{\tilde{f}, \text{FI-1loop}}^2|_R + m_{\tilde{f}, \text{FI-1loop}}^2|_{\tilde{F}}. \quad (89)$$

$U(1)_Y$ FI D -term is also generated at two loops through the diagram shown in Fig. 16. The resulting light sfermion masses from extra-matter loops are written as

$$\begin{aligned} m_{\tilde{f}, \text{FI-2loop}}^2|_R &= \sum_A g_Y^2 g_A^2 Y_{\tilde{f}} \text{Tr}_R \left[Y_R C_R^A \left\{ \sum_{\alpha} (V_{\alpha 1} V_{1\alpha}^\dagger - V_{\alpha 2} V_{2\alpha}^\dagger) I_1(m_{\alpha}^2) \right. \right. \\ &\quad + 4 \sum_{\alpha} (V_{\alpha 1} V_{1\alpha}^\dagger - V_{\alpha 2} V_{2\alpha}^\dagger) I_2(m_{\alpha}^2) \\ &\quad - \sum_{\alpha\beta\gamma} (V_{\alpha 1} V_{1\beta}^\dagger - V_{\alpha 2} V_{2\beta}^\dagger) (V_{\beta 1} V_{1\gamma}^\dagger - V_{\beta 2} V_{2\gamma}^\dagger) \\ &\quad \times (V_{\gamma 1} V_{1\alpha}^\dagger - V_{\gamma 2} V_{2\alpha}^\dagger) I_3(m_{\alpha}^2, m_{\beta}^2, m_{\gamma}^2) \\ &\quad \left. \left. + \sum_{\alpha\beta} (V_{\alpha 1} V_{1\beta}^\dagger - V_{\alpha 2} V_{2\beta}^\dagger) (\delta_{m^2})_{\beta\alpha} I_c(m_{\alpha}^2, m_{\beta}^2) \right\} \right], \end{aligned} \quad (90)$$

in the Feynman gauge. Here, functions I 's are defined as

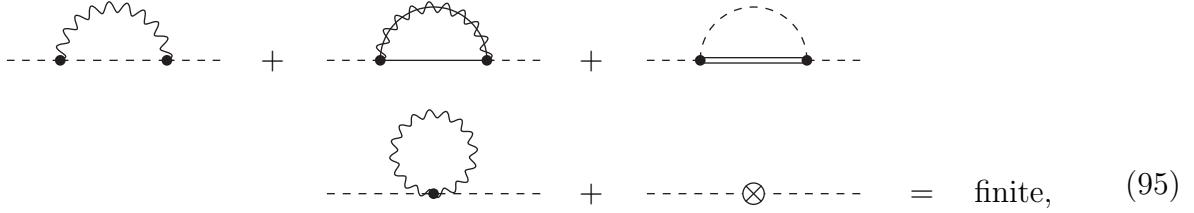
$$I_1(m_{\alpha}^2) = \int \frac{d^4 p}{(2\pi)^4} \int \frac{d^4 k}{(2\pi)^4} \frac{1}{(p^2 - m_{\alpha}^2)^2} \frac{(2p - k)^2}{k^2} \frac{1}{(p - k)^2 - m_{\alpha}^2}, \quad (91)$$

$$I_2(m_\alpha^2) = \int \frac{d^4 p}{(2\pi)^4} \int \frac{d^4 k}{(2\pi)^4} \frac{1}{(p^2 - m_\alpha^2)^2} \frac{k^2 - k \cdot p}{k^2} \frac{1}{(p - k)^2 - m_\psi^2}, \quad (92)$$

$$I_3(m_\alpha^2, m_\beta^2, m_\gamma^2) = \int \frac{d^4 p}{(2\pi)^4} \frac{1}{p^2 - m_\alpha^2} \frac{1}{p^2 - m_\beta^2} \int \frac{d^4 q}{(2\pi)^4} \frac{1}{q^2 - m_\gamma^2}, \quad (93)$$

$$I_c(m_\alpha^2, m_\beta^2) = i \int \frac{d^4 p}{(2\pi)^4} \frac{1}{p^2 - m_\alpha^2} \frac{1}{p^2 - m_\beta^2}. \quad (94)$$

The counterterm $(\delta_{m^2})_{\alpha\beta}$ for the extra-scalar masses, $\mathcal{L} = -\sum_A g_A^2 C_R^A (\delta_{m^2})_{\alpha\beta} (\tilde{\psi}_{\text{ex}})_\alpha^* (\tilde{\psi}_{\text{ex}})_\beta$, is determined to cancel the divergence,



as

$$(\delta_{m^2})_{\alpha\beta} = \frac{1}{(4\pi)^2} \left(\frac{1}{\epsilon} \right) \times \begin{pmatrix} -\sin^2 2\theta (m_1^2 - m_2^2) - 4m_\psi^2 & \sin 2\theta \cos 2\theta (m_1^2 - m_2^2) e^{i\alpha} \\ \sin 2\theta \cos 2\theta (m_1^2 - m_2^2) e^{-i\alpha} & \sin^2 2\theta (m_1^2 - m_2^2) - 4m_\psi^2 \end{pmatrix}. \quad (96)$$

Substituting Eq. (96) into Eq. (90), we obtain

$$\begin{aligned} m_{\tilde{f}, \text{FI-2loop}}^2 |_R = & -2 \sum_A \left(\frac{\alpha_Y}{4\pi} \right) \left(\frac{\alpha_A}{4\pi} \right) Y_{\tilde{f}} \text{Tr}_R \left[Y_R C_R^A \cos 2\theta (m_1^2 - m_2^2) \right] \left(\frac{1}{\epsilon} + 3 \right) \\ & + 4 \sum_A \left(\frac{\alpha_Y}{4\pi} \right) \left(\frac{\alpha_A}{4\pi} \right) Y_{\tilde{f}} \text{Tr}_R \left[Y_R C_R^A \cos 2\theta \left\{ -\frac{1}{2} m_\psi^2 \log^2 m_1^2 + \frac{1}{2} m_\psi^2 \log^2 m_2^2 \right. \right. \\ & \quad \left. \left. + (m_1^2 + m_\psi^2) \log m_1^2 - (m_2^2 + m_\psi^2) \log m_2^2 \right. \right. \\ & \quad \left. \left. + (m_1^2 - m_\psi^2) \text{Li}_2(1 - \frac{m_\psi^2}{m_1^2}) - (m_2^2 - m_\psi^2) \text{Li}_2(1 - \frac{m_\psi^2}{m_2^2}) \right\} \right. \\ & \quad \left. + \frac{1}{4} Y_R C_R^A \cos 2\theta \sin^2 2\theta \left\{ 2 \frac{(m_1^2 \log m_1^2 - m_2^2 \log m_2^2)^2}{m_1^2 - m_2^2} \right. \right. \\ & \quad \left. \left. - 4(m_1^2 \log m_1^2 - m_2^2 \log m_2^2) - (m_1^2 \log^2 m_1^2 - m_2^2 \log^2 m_2^2) \right. \right. \\ & \quad \left. \left. + (m_1^2 - m_2^2)(2 + \log m_1^2 + \log m_2^2 - \log m_1^2 \log m_2^2) \right\} \right]. \end{aligned} \quad (97)$$

This contribution vanishes when $\theta = \pm\pi/4$ as it should be. The contribution from the heavy

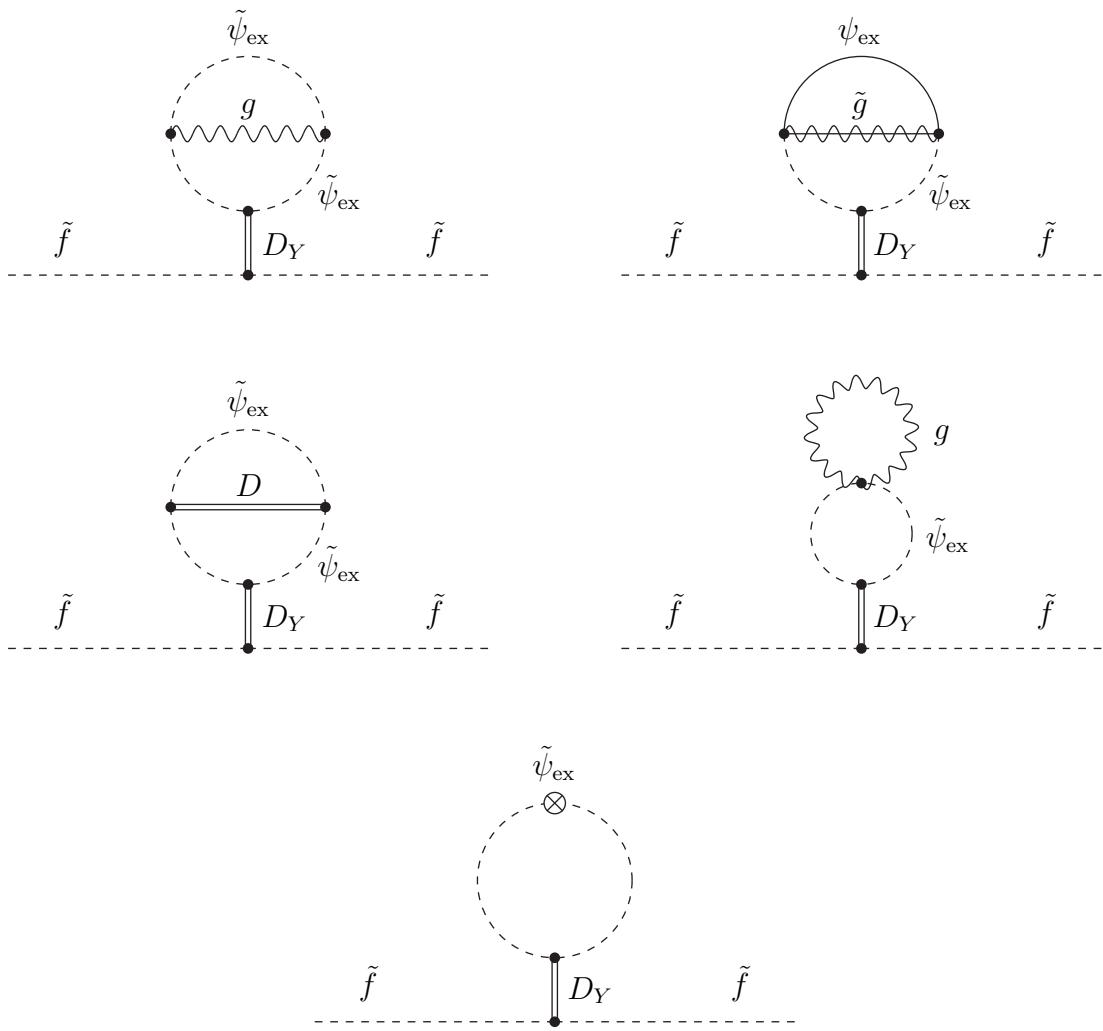


Figure 16: Two-loop diagram contributing to the light sfermion masses through the generation of $U(1)_Y$ FI D -term.

sfermions is read off by taking the limit $m_\psi \rightarrow 0$ and $\theta \rightarrow 0$ as

$$\begin{aligned} m_{\tilde{f}, \text{FI-2loop}}^2|_{\tilde{F}} = & -2 \sum_A \left(\frac{\alpha_Y}{4\pi} \right) \left(\frac{\alpha_A}{4\pi} \right) Y_{\tilde{f}} \text{Tr}_{\tilde{F}} \left[Y_{\tilde{F}} C_{\tilde{F}}^A m_{\tilde{F}}^2 \right] \left(\frac{1}{\epsilon} + 3 \right) \\ & + 4 \sum_A \left(\frac{\alpha_Y}{4\pi} \right) \left(\frac{\alpha_A}{4\pi} \right) Y_{\tilde{f}} \text{Tr}_{\tilde{F}} \left[Y_{\tilde{F}} C_{\tilde{F}}^A m_{\tilde{F}}^2 \left(\log m_{\tilde{F}}^2 + \frac{1}{6}\pi^2 \right) \right], \end{aligned} \quad (98)$$

reproducing the earlier result derived in Ref. [12].⁸ In addition, there is another diagram which contributes to the light sfermion masses at the two-loop level. The diagram is shown in Fig. 17 and its contribution is

$$m_{\tilde{f}, \text{FI-2(1loop)}}^2 = -g_Y^4 Y_{\tilde{f}} J_1 J_2 + i g_Y^4 Y_{\tilde{f}} \delta_{m_Y^2} J_2 + i g_Y^4 Y_{\tilde{f}} J_1 \delta_{g_Y^4}, \quad (99)$$

where

$$\begin{aligned} J_1 = & \text{Tr}_R \left[Y_R \sum_\alpha (V_{\alpha 1} V_{1\alpha}^\dagger - V_{\alpha 2} V_{2\alpha}^\dagger) \int \frac{d^4 p}{(2\pi)^4} \frac{1}{p^2 - m_\alpha^2} \right] \\ & + \text{Tr}_{\tilde{F}} \left[Y_{\tilde{F}} \int \frac{d^4 p}{(2\pi)^4} \frac{1}{p^2 - m_{\tilde{F}}^2} \right], \end{aligned} \quad (100)$$

$$\begin{aligned} J_2 = & \text{Tr}_R \left[Y_R^2 \sum_{\gamma\delta} (V_{\gamma 1} V_{1\delta}^\dagger - V_{\gamma 2} V_{2\delta}^\dagger) (V_{\delta 1} V_{1\gamma}^\dagger - V_{\delta 2} V_{2\gamma}^\dagger) \int \frac{d^4 q}{(2\pi)^4} \frac{1}{q^2 - m_\gamma^2} \frac{1}{q^2 - m_\delta^2} \right] \\ & + \text{Tr}_{\tilde{F}} \left[Y_{\tilde{F}}^2 \int \frac{d^4 q}{(2\pi)^4} \frac{1}{(q^2 - m_{\tilde{F}}^2)^2} \right] + \text{Tr}_{\tilde{f}} \left[Y_{\tilde{f}}^2 \int \frac{d^4 q}{(2\pi)^4} \frac{1}{(q^2 - m_{\tilde{f}}^2)^2} \right]. \end{aligned} \quad (101)$$

The counterterms $\delta_{m_Y^2}$ and $\delta_{g_Y^4}$ are defined by

$$\begin{aligned} \mathcal{L} = & -g_Y^2 \delta_{m_Y^2} \left\{ \sum_{\alpha\beta} (V_{\alpha 1} V_{1\beta}^\dagger - V_{\alpha 2} V_{2\beta}^\dagger) (\tilde{\psi}_{\text{ex}})_\alpha^* Y_R (\tilde{\psi}_{\text{ex}})_\beta + \tilde{\psi}_{\tilde{F}}^* Y_{\tilde{F}} \tilde{\psi}_{\tilde{F}} + \tilde{f}^* Y_{\tilde{f}} \tilde{f} \right\} \\ & -g_Y^4 \delta_{g_Y^4} \tilde{f}^* Y_{\tilde{f}} \tilde{f} \left\{ \sum_{\alpha\beta} (V_{\alpha 1} V_{1\beta}^\dagger - V_{\alpha 2} V_{2\beta}^\dagger) (\tilde{\psi}_{\text{ex}})_\alpha^* Y_R (\tilde{\psi}_{\text{ex}})_\beta + \tilde{\psi}_{\tilde{F}}^* Y_{\tilde{F}} \tilde{\psi}_{\tilde{F}} \right\}. \end{aligned} \quad (102)$$

They are determined by the conditions,



$$\text{---} \cdot \text{---} + \text{---} \otimes \text{---} = \text{finite}, \quad (103)$$

⁸ Their result is different from ours by a finite part proportional to $-\gamma + \log(4\pi)$, since they subtracted divergences not in the $\overline{\text{MS}}$ scheme.

and

$$= \text{finite}, \quad (104)$$

respectively, which lead to

$$\delta_{m_Y^2} = \frac{1}{(4\pi)^2} \left\{ \text{Tr}_R \left[Y_R \cos 2\theta (m_1^2 - m_2^2) \right] + \text{Tr}_{\tilde{F}} \left[Y_{\tilde{F}} m_{\tilde{F}}^2 \right] \right\} \left(\frac{1}{\epsilon} \right), \quad (105)$$

$$\delta_{g_Y^4} = \frac{1}{(4\pi)^2} \left\{ 2 \text{Tr}_R \left[Y_R^2 \right] + \text{Tr}_{\tilde{F}} \left[Y_{\tilde{F}}^2 \right] + \text{Tr}_{\tilde{f}} \left[Y_{\tilde{f}}^2 \right] \right\} \left(\frac{1}{\epsilon} \right). \quad (106)$$

From Eqs. (99, 105, 106), we obtain

$$\begin{aligned} m_{\tilde{f}, \text{FI-2loop}}^2 &= \\ &- \left(\frac{\alpha_Y}{4\pi} \right)^2 Y_{\tilde{f}} \left(\frac{1}{\epsilon} \right)^2 \\ &\times \left\{ \text{Tr}_R \left[Y_R \cos 2\theta (m_1^2 - m_2^2) \right] + \text{Tr}_{\tilde{F}} \left[Y_{\tilde{F}} m_{\tilde{F}}^2 \right] \right\} \left\{ 2 \text{Tr}_R \left[Y_R^2 \right] + \text{Tr}_{\tilde{F}} \left[Y_{\tilde{F}}^2 \right] + \text{Tr}_{\tilde{f}} \left[Y_{\tilde{f}}^2 \right] \right\} \\ &- \left(\frac{\alpha_Y}{4\pi} \right)^2 Y_{\tilde{f}} \left\{ \text{Tr}_R \left[Y_R \cos 2\theta (m_1^2 - m_2^2) \right] + \text{Tr}_{\tilde{F}} \left[Y_{\tilde{F}} m_{\tilde{F}}^2 \right] \right. \\ &\quad \left. - \text{Tr}_R \left[Y_R \cos 2\theta (m_1^2 \log m_1^2 - m_2^2 \log m_2^2) \right] - \text{Tr}_{\tilde{F}} \left[Y_{\tilde{F}} m_{\tilde{F}}^2 \log m_{\tilde{F}}^2 \right] \right\} \\ &\times \left\{ \text{Tr}_R \left[Y_R^2 \cos^2 2\theta (\log m_1^2 + \log m_2^2) - 2Y_R^2 \sin^2 2\theta \left(1 - \frac{m_1^2 \log m_1^2 - m_2^2 \log m_2^2}{m_1^2 - m_2^2} \right) \right] \right. \\ &\quad \left. + \text{Tr}_{\tilde{F}} \left[Y_{\tilde{F}}^2 \log m_{\tilde{F}}^2 \right] + \text{Tr}_{\tilde{f}} \left[Y_{\tilde{f}}^2 \log m_{\tilde{f}}^2 \right] \right\}. \end{aligned} \quad (107)$$

Thus, the light sfermion masses induced at two loops through the generation of $U(1)_Y$ FI D -term are given by

$$m_{\tilde{f}, \text{FI-2loop}}^2 = m_{\tilde{f}, \text{FI-2loop}}^2|_R + m_{\tilde{f}, \text{FI-2loop}}^2|_{\tilde{F}} + m_{\tilde{f}, \text{FI-2loop}}^2. \quad (108)$$

Altogether, the light sfermion masses at the two-loop level are given by

$$m_{\tilde{f}, \text{total}}^2 = m_{\tilde{f}, \text{direct}}^2 + m_{\tilde{f}, \text{FI-1loop}}^2 + m_{\tilde{f}, \text{FI-2loop}}^2, \quad (109)$$

combining Eqs. (86, 89, 108).

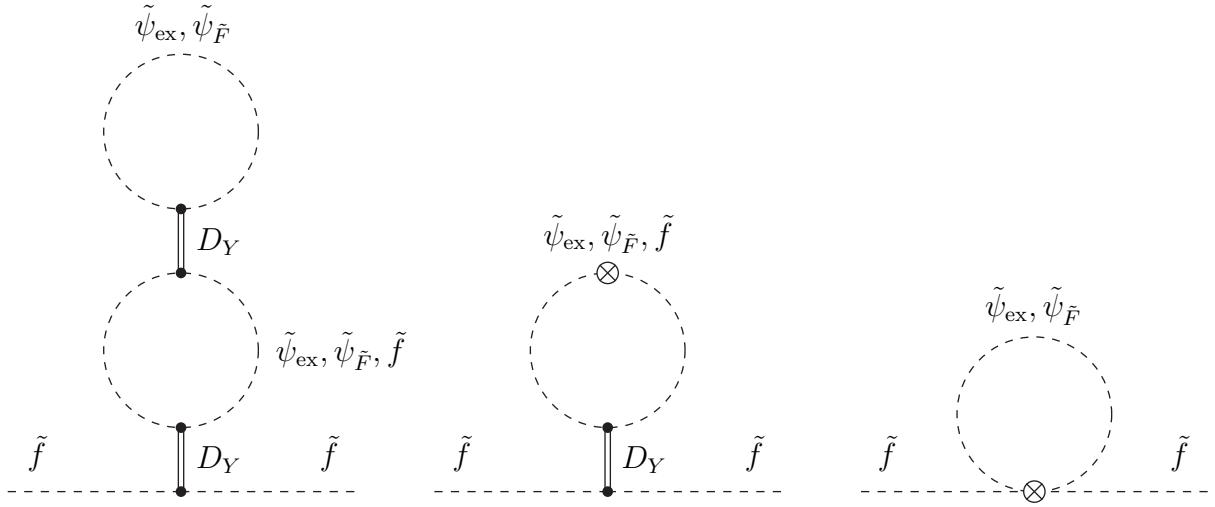


Figure 17: Two-loop diagram contributing to the light sfermion masses.

B.3 The finite case

In the previous subsection, we have calculated the gaugino and the light sfermion masses at the two-loop level in the $\overline{\text{DR}}'$ scheme. The light sfermion masses generically have divergent contribution, so that they receive large negative contribution from the RG evolution. However, the divergences are canceled among loops of various heavy particles under the conditions Eqs. (3, 4, 5) discussed in the text. These conditions are written as

$$\text{Tr}_R \left[T_R^A m_\psi^2 (y_1 + y_2 - 2) \right] + \text{Tr}_{\tilde{F}} \left[T_{\tilde{F}}^A m_{\tilde{F}}^2 \right] = 0, \quad (110)$$

$$\text{Tr}_R \left[Y_R \cos 2\theta (m_1^2 - m_2^2) \right] + \text{Tr}_{\tilde{F}} \left[Y_{\tilde{F}} m_{\tilde{F}}^2 \right] = 0, \quad (111)$$

$$\text{Tr}_R \left[Y_R C_R^A \cos 2\theta (m_1^2 - m_2^2) \right] + \text{Tr}_{\tilde{F}} \left[Y_{\tilde{F}} C_{\tilde{F}}^A m_{\tilde{F}}^2 \right] = 0, \quad (112)$$

in the notation of this appendix. Using these relations in Eqs. (86, 89, 108), we obtain the finite contribution to the light sfermion masses,

$$m_{\tilde{f}, \text{total}}^2 = m_{\tilde{f}, \text{direct}}^2 + m_{\tilde{f}, \text{FI-1loop}}^2 + m_{\tilde{f}, \text{FI-2loop}}^2; \quad (113)$$

$$m_{\tilde{f}, \text{direct}}^2 = 4 \sum_A \left(\frac{\alpha_A}{4\pi} \right)^2 C_{\tilde{f}}^A \text{Tr}_R \left[T_R^A m_\psi^2 \left\{ (y_1 + y_2 - 2) \log m_\psi^2 + (y_1 \log y_1 + y_2 \log y_2) \right\} \right]$$

$$\begin{aligned}
& -2 \left(y_1 \text{Li}_2(1 - \frac{1}{y_1}) + y_2 \text{Li}_2(1 - \frac{1}{y_2}) \right) + \frac{1}{2} \sin^2 2\theta \left(y_1 \text{Li}_2(1 - \frac{y_2}{y_1}) + y_2 \text{Li}_2(1 - \frac{y_1}{y_2}) \right) \Big\} \\
& + 4 \sum_A \left(\frac{\alpha_A}{4\pi} \right)^2 C_{\tilde{f}}^A \text{Tr}_{\tilde{F}} \left[T_{\tilde{F}}^A m_{\tilde{F}}^2 \left(\log m_{\tilde{F}}^2 - \frac{1}{3}\pi^2 \right) \right], \tag{114}
\end{aligned}$$

$$\begin{aligned}
m_{\tilde{f}, \text{FI-1loop}}^2 = & \\
& \left(\frac{\alpha_Y}{4\pi} \right) Y_{\tilde{f}} \text{Tr}_R \left[Y_R \cos 2\theta \left(m_1^2 \log m_1^2 - m_2^2 \log m_2^2 \right) \right] \\
& + \left(\frac{\alpha_Y}{4\pi} \right) Y_{\tilde{f}} \text{Tr}_{\tilde{F}} \left[Y_{\tilde{F}} m_{\tilde{F}}^2 \log m_{\tilde{F}}^2 \right], \tag{115}
\end{aligned}$$

$$\begin{aligned}
m_{\tilde{f}, \text{FI-2loop}}^2 = & \\
& 4 \sum_A \left(\frac{\alpha_Y}{4\pi} \right) \left(\frac{\alpha_A}{4\pi} \right) Y_{\tilde{f}} \text{Tr}_R \left[Y_R C_R^A \cos 2\theta \left\{ -\frac{1}{2} m_\psi^2 \log^2 m_1^2 + \frac{1}{2} m_\psi^2 \log^2 m_2^2 \right. \right. \\
& + (m_1^2 + m_\psi^2) \log m_1^2 - (m_2^2 + m_\psi^2) \log m_2^2 \\
& + (m_1^2 - m_\psi^2) \text{Li}_2(1 - \frac{m_\psi^2}{m_1^2}) - (m_2^2 - m_\psi^2) \text{Li}_2(1 - \frac{m_\psi^2}{m_2^2}) \Big\} \\
& + \frac{1}{4} Y_R C_R^A \cos 2\theta \sin^2 2\theta \left\{ 2 \frac{(m_1^2 \log m_1^2 - m_2^2 \log m_2^2)^2}{m_1^2 - m_2^2} \right. \\
& - 4(m_1^2 \log m_1^2 - m_2^2 \log m_2^2) - (m_1^2 \log^2 m_1^2 - m_2^2 \log^2 m_2^2) \\
& \left. \left. + (m_1^2 - m_2^2)(2 + \log m_1^2 + \log m_2^2 - \log m_1^2 \log m_2^2) \right\} \right] \\
& + 4 \sum_A \left(\frac{\alpha_Y}{4\pi} \right) \left(\frac{\alpha_A}{4\pi} \right) Y_{\tilde{f}} \text{Tr}_{\tilde{F}} \left[Y_{\tilde{F}} C_{\tilde{F}}^A m_{\tilde{F}}^2 \left(\log m_{\tilde{F}}^2 + \frac{1}{6}\pi^2 \right) \right] \\
& + \left(\frac{\alpha_Y}{4\pi} \right)^2 Y_{\tilde{f}} \left\{ \text{Tr}_R \left[Y_R \cos 2\theta \left(m_1^2 \log m_1^2 - m_2^2 \log m_2^2 \right) \right] + \text{Tr}_{\tilde{F}} \left[Y_{\tilde{F}} m_{\tilde{F}}^2 \log m_{\tilde{F}}^2 \right] \right\} \\
& \times \left\{ \text{Tr}_R \left[Y_R^2 \cos^2 2\theta \left(\log m_1^2 + \log m_2^2 \right) - 2Y_R^2 \sin^2 2\theta \left(1 - \frac{m_1^2 \log m_1^2 - m_2^2 \log m_2^2}{m_1^2 - m_2^2} \right) \right] \right. \\
& \left. + \text{Tr}_{\tilde{F}} \left[Y_{\tilde{F}}^2 \log m_{\tilde{F}}^2 \right] \right\}. \tag{116}
\end{aligned}$$

It should be understood that the coupling constants α_A and α_Y are the renormalized ones at the scale μ , $\alpha_A(\mu)$ and $\alpha_Y(\mu)$, and the logarithms $\log m^2$ of various masses are normalized at the scale μ , $\log(m^2/\mu^2)$. Here, we have dropped the term proportional to $\text{Tr}_{\tilde{f}}[Y_{\tilde{f}}^2 \log m_{\tilde{f}}^2]$ which is the contribution from the RG evolution below μ . Note that the contribution $m_{\tilde{f}, \text{FI-1loop}}^2$ from one-loop U(1)_Y FI D -term is generically nonvanishing even if we assign U(1)_X charges consistent with the SU(5)_{GUT}, since the extra scalars embedded in a common SU(5)_{GUT} multiplet could have different masses due to the difference between their supersymmetric masses caused by RG evolution below the GUT scale.

Furthermore, the above expression is considerably simplified when the following two conditions are satisfied:

1. The $U(1)_X$ charge assignment is consistent with $SU(5)_{\text{GUT}}$: $\text{Tr}_{\tilde{F}} \left[Y_{\tilde{F}} f(m_{\tilde{F}}^2) \right] = 0$
2. The extra-matter multiplets have an invariance under the parity $\Psi_{\text{ex}} \leftrightarrow \bar{\Psi}_{\text{ex}}$: $\theta = \pm \frac{\pi}{4}$

These conditions are satisfied in the case of Model (I) and Model (II) considered in the text. Then, Eqs. (114, 115, 116) are reduced to

$$\begin{aligned} m_{\tilde{f}, \text{direct}}^2 &= 4 \sum_A \left(\frac{\alpha_A}{4\pi} \right)^2 C_{\tilde{f}}^A \text{Tr}_R \left[T_R^A m_\psi^2 \left\{ (y_1 + y_2 - 2) \log m_\psi^2 + (y_1 \log y_1 + y_2 \log y_2) \right. \right. \\ &\quad \left. \left. - 2 \left(y_1 \text{Li}_2(1 - \frac{1}{y_1}) + y_2 \text{Li}_2(1 - \frac{1}{y_2}) \right) + \frac{1}{2} \left(y_1 \text{Li}_2(1 - \frac{y_2}{y_1}) + y_2 \text{Li}_2(1 - \frac{y_1}{y_2}) \right) \right\} \right] \\ &\quad + 4 \sum_A \left(\frac{\alpha_A}{4\pi} \right)^2 C_{\tilde{f}}^A \text{Tr}_{\tilde{F}} \left[T_{\tilde{F}}^A m_{\tilde{F}}^2 \left(\log m_{\tilde{F}}^2 - \frac{1}{3} \pi^2 \right) \right], \end{aligned} \quad (117)$$

$$m_{\tilde{f}, \text{FI-1loop}}^2 = 0, \quad (118)$$

$$m_{\tilde{f}, \text{FI-2loop}}^2 = 4 \sum_A \left(\frac{\alpha_Y}{4\pi} \right) \left(\frac{\alpha_A}{4\pi} \right) Y_{\tilde{f}} \text{Tr}_{\tilde{F}} \left[Y_{\tilde{F}} C_{\tilde{F}}^A m_{\tilde{F}}^2 \log m_{\tilde{F}}^2 \right]. \quad (119)$$

C Constraints from Δm_K and ϵ_K

In this appendix, we calculate the constraints on the first-two generation sfermion masses from Δm_K and ϵ_K in effective SUSY, where the gluino is much lighter than the first-two generation sfermions. In the following calculation, we only consider the gluino box diagram since it gives a dominant contribution. According to Refs. [24, 25, 26], we not only take into account the leading QCD corrections but also make use of B parameters instead of the vacuum insertion approximation.

The mass matrix for the down-type squark is relevant to the gluino box diagram. In a basis where the down-type Yukawa matrix is diagonal, the mass matrix is

$$\mathcal{L}_{\text{mass}} = - \begin{pmatrix} \tilde{d}_{L_i}^* & \tilde{d}_{R_j}^* \end{pmatrix} \begin{pmatrix} (\mathcal{M}_{LL}^2)_{ij} & (\mathcal{M}_{LR}^2)_{ij} \\ (\mathcal{M}_{LR}^{2\dagger})_{ij} & (\mathcal{M}_{RR}^2)_{ij} \end{pmatrix} \begin{pmatrix} \tilde{d}_{L_j} \\ \tilde{d}_{R_j} \end{pmatrix}, \quad (120)$$

where the subscripts i, j are the indices of the generation. We here restrict the subscript i to $i = 1, 2$, since we calculate the constraints on the first-two generation sfermion masses. From now we take the left-right mixing mass term $\mathcal{M}_{LR}^2 = \mathbf{0}$ because of the small Yukawa couplings. In consequence, \mathcal{M}_{LL}^2 and \mathcal{M}_{RR}^2 are diagonalized separately by the sfermion mixing matrices $(U^L)_{X_L, i}$ and $(U^R)_{X_R, i}$ as follows:

$$U^L \mathcal{M}_{LL}^2 U^{L\dagger} = \text{diag} \left(m_{X_L}^2 \right), \quad U^R \mathcal{M}_{RR}^2 U^{R\dagger} = \text{diag} \left(m_{X_R}^2 \right), \quad (121)$$

where $X_L = 1_L, 2_L$ and $X_R = 1_R, 2_R$.

The $\Delta S = 2$ effective Lagrangian at the scale m_D , where the heavy sfermions decouple, is written as

$$\mathcal{L}_{\text{eff}} = \alpha_3^2(m_D) \left[C_1 \mathcal{O}_1 + \tilde{C}_1 \tilde{\mathcal{O}}_1 + C_4 \mathcal{O}_4 + C_5 \mathcal{O}_5 \right]. \quad (122)$$

The operators $\mathcal{O}_{1,4,5}$ and their coefficients $C_{1,4,5}$ are defined as follows:

$$\mathcal{O}_1 = (\bar{d}_\alpha \gamma_\mu P_L s_\alpha) (\bar{d}_\beta \gamma^\mu P_L s_\beta), \quad (123)$$

$$\mathcal{O}_4 = (\bar{d}_\alpha P_L s_\alpha) (\bar{d}_\beta P_R s_\beta), \quad (124)$$

$$\mathcal{O}_5 = (\bar{d}_\alpha P_L s_\beta) (\bar{d}_\beta P_R s_\alpha), \quad (125)$$

$$C_1 = U_{1,X_L}^{L\dagger} U_{X_L,2}^L U_{1,Y_L}^{L\dagger} U_{Y_L,2}^L \left(\frac{1}{9} I_{X_L,Y_L} + \frac{11}{36} \tilde{I}_{X_L,Y_L} \right), \quad (126)$$

$$C_4 = U_{1,X_L}^{L\dagger} U_{X_L,2}^L U_{1,Y_R}^{R\dagger} U_{Y_R,2}^R \left(\frac{7}{3} I_{X_L,Y_R} - \frac{1}{3} \tilde{I}_{X_L,Y_R} \right), \quad (127)$$

$$C_5 = U_{1,X_L}^{L\dagger} U_{X_L,2}^L U_{1,Y_R}^{R\dagger} U_{Y_R,2}^R \left(\frac{1}{9} I_{X_L,Y_R} + \frac{5}{9} \tilde{I}_{X_L,Y_R} \right). \quad (128)$$

$\tilde{\mathcal{O}}_1$ and \tilde{C}_1 are obtained with the replacement $L \rightarrow R$ in \mathcal{O}_1 and C_1 , respectively. The loop integrals $I_{X,Y}$ and $\tilde{I}_{X,Y}$ in the above equations are

$$I_{X,Y} = 0, \quad \tilde{I}_{X,Y} = -\frac{\log m_X^2 - \log m_Y^2}{m_X^2 - m_Y^2}, \quad (129)$$

where we have neglected the contributions dependent on the gluino mass $m_{\tilde{g}}$, since they are suppressed by $m_{\tilde{g}}^2/m_X^2$ ($\ll 1$).

The above effective Lagrangian is valid for an arbitrary down-type squark mass matrix. We here give a convenient expression by using the so-called mass insertion method [33]. We introduce two parameters essential for the mass insertion method. One is the averaged mass \bar{m}_{LL}^2 (\bar{m}_{RR}^2) for the left- (right-) handed down-type squarks in the first-two generations, which is defined by a geometric mean as

$$\bar{m}_{LL}^2 \equiv \sqrt{m_{1_L}^2 m_{2_L}^2}, \quad \bar{m}_{RR}^2 \equiv \sqrt{m_{1_R}^2 m_{2_R}^2}. \quad (130)$$

The other is the off-diagonal element δ_{LL} (δ_{RR}) of \mathcal{M}_{LL}^2 (\mathcal{M}_{RR}^2), which is normalized with the averaged mass as

$$\delta_{LL} \equiv \frac{(\mathcal{M}_{LL}^2)_{1L,2L}}{\bar{m}_{LL}^2}, \quad \delta_{RR} \equiv \frac{(\mathcal{M}_{RR}^2)_{1R,2R}}{\bar{m}_{RR}^2}. \quad (131)$$

By taking a leading order of the mass differences, $(m_{1_L}^2 - m_{2_L}^2)$ and $(m_{1_R}^2 - m_{2_R}^2)$, the coefficients in Eqs. (126, 127, 128) become

$$C_1 = -\frac{11}{108} \frac{\delta_{LL}^2}{\bar{m}_{LL}^2}, \quad (132)$$

$$C_4 = \frac{1}{9} \frac{\delta_{LL}\delta_{RR}}{\bar{m}_{LL}\bar{m}_{RR}}, \quad (133)$$

$$C_5 = -\frac{5}{27} \frac{\delta_{LL}\delta_{RR}}{\bar{m}_{LL}\bar{m}_{RR}}. \quad (134)$$

\tilde{C}_1 is obtained with the replacement $L \rightarrow R$ in C_1 . In Section 4, we often use the expression in the mass insertion method in estimating the bounds on the light sfermion masses. However, Figs. 6-11 are obtained by making use of the exact coefficients given in Eqs. (126, 127, 128).

Since the above effective Lagrangian is obtained at the heavy-sfermion mass scale m_D , we must evolve it using RG equations to the hadronic scale μ_{had} , where hadronic matrix elements are evaluated. The coefficients at μ_{had} are calculated at the one-loop level as follows [24]:

$$C_1(\mu_{\text{had}}) = \kappa_1 C_1(m_D), \quad (135)$$

$$\tilde{C}_1(\mu_{\text{had}}) = \kappa_1 \tilde{C}_1(m_D), \quad (136)$$

$$C_4(\mu_{\text{had}}) = \kappa_4 C_4(m_D) + \frac{1}{3} (\kappa_4 - \kappa_5) C_5(m_D), \quad (137)$$

$$C_5(\mu_{\text{had}}) = \kappa_5 C_5(m_D), \quad (138)$$

where

$$\kappa_1 = \left(\frac{\alpha_3(m_b)}{\alpha_3(\mu_{\text{had}})} \right)^{6/25} \left(\frac{\alpha_3(m_t)}{\alpha_3(m_b)} \right)^{6/23} \left(\frac{\alpha_3(m_{\tilde{g}})}{\alpha_3(m_t)} \right)^{6/21} \left(\frac{\alpha_3(m_D)}{\alpha_3(m_{\tilde{g}})} \right)^{-2/b_3}, \quad (139)$$

$$\kappa_4 = \kappa_1^{-4}, \quad (140)$$

$$\kappa_5 = \kappa_1^{1/2}. \quad (141)$$

Here, we have taken the hadronic scale $\mu_{\text{had}} = 2$ GeV according to Ref. [25] and assumed that all the light sfermions and the gauginos decouple at the gluino mass scale for simplicity. b_3 is the coefficient of the one-loop beta function for the strong coupling between the gluino mass $m_{\tilde{g}}$ and the heavy-sfermion mass scale m_D .

Instead of using the vacuum insertion approximation, we represent the hadronic matrix elements of the renormalized operators $\mathcal{O}(\mu)$ at the renormalization scale μ in terms of the corresponding B parameters as follows:

$$\langle K^0 | \mathcal{O}_1(\mu) | \bar{K}^0 \rangle = \langle K^0 | \tilde{\mathcal{O}}_1(\mu) | \bar{K}^0 \rangle = \frac{1}{3} m_K f_K^2 B_1(\mu), \quad (142)$$

$$\langle K^0 | \mathcal{O}_4(\mu) | \bar{K}^0 \rangle = \frac{1}{4} \left(\frac{m_K}{m_s(\mu) + m_d(\mu)} \right)^2 m_K f_K^2 B_4(\mu), \quad (143)$$

$$\langle K^0 | \mathcal{O}_5(\mu) | \bar{K}^0 \rangle = \frac{1}{12} \left(\frac{m_K}{m_s(\mu) + m_d(\mu)} \right)^2 m_K f_K^2 B_5(\mu), \quad (144)$$

where $m_K = 497.7$ MeV, $f_K = 160$ MeV, $m_s(2 \text{ GeV}) = 125$ MeV and $m_d(2 \text{ GeV}) = 7$ MeV. In our calculation, we use the following values for the B parameters obtained by lattice calculations at $\mu = 2$ GeV [34, 35]:

$$B_1(\mu = 2 \text{ GeV}) = 0.60(6), \quad (145)$$

$$B_4(\mu = 2 \text{ GeV}) = 1.03(6), \quad (146)$$

$$B_5(\mu = 2 \text{ GeV}) = 0.73(10). \quad (147)$$

We have now explained all the elements necessary for calculating the SUSY contribution to the $K_L - K_S$ mass difference,

$$\Delta m_{K,\text{SUSY}} = -2 \operatorname{Re} \langle K^0 | \mathcal{L}_{\text{eff}} | \bar{K}^0 \rangle. \quad (148)$$

Using them, we obtain constraints from Δm_K by imposing a condition that the SUSY contribution does not saturate the experimental value,

$$2|\langle K^0 | \mathcal{L}_{\text{eff}} | \bar{K}^0 \rangle| < \Delta m_K = 3.49 \times 10^{-15} \text{ GeV}, \quad (149)$$

in the case where the δ_{LL} and δ_{RR} have no CP-violating phases. The constraints can be expressed in a simple form using the mass insertion as follows:

$$\delta_{LL, RR} < \frac{\bar{m}_{LL, RR}}{(25 \sim 35) \text{ TeV}}, \quad (150)$$

$$(\delta_{LL}\delta_{RR})^{1/2} < \frac{(\bar{m}_{LL}\bar{m}_{RR})^{1/2}}{(150 \sim 250) \text{ TeV}}. \quad (151)$$

If δ_{LL} and/or δ_{RR} have CP-violating phases, there is another constraint from ϵ_K ,

$$\frac{1}{\sqrt{2}\Delta m_K} |\operatorname{Im} \langle K^0 | \mathcal{L}_{\text{eff}} | \bar{K}^0 \rangle| < \epsilon_K = 2.3 \times 10^{-3}. \quad (152)$$

The constraint is the severest when $\langle K^0 | \mathcal{L}_{\text{eff}} | \bar{K}^0 \rangle$ is pure imaginary, and then the above constraint Eq. (152) is rewritten as

$$2|\langle K^0 | \mathcal{L}_{\text{eff}} | \bar{K}^0 \rangle| < 2\sqrt{2}\epsilon_K\Delta m_K = 6.5 \times 10^{-3}\Delta m_K. \quad (153)$$

Thus, the constraints from ϵ_K can be severer than those from Δm_K by a factor of $(2\sqrt{2}\epsilon_K)^{-1/2} \sim 12.4$.

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